OXFORD IB COURSE PREPARATION

MATHEMATICS

FOR IB DIPLOMA COURSE PREPARATION ANSWERS

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ANSWERS

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OXFORD UNIVERSITY PRESS

Exercise 1.1

1 $\mathbb{N}$ **2** $\mathbb{Z}$ **3** $\mathbb{N}$ **4** $\mathbb{R}$ **5** $\mathbb{N}$

6 $\mathbb{Q}$ **7** $\mathbb{Z}$

Exercise 1.2

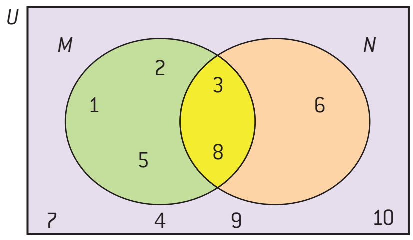
1 a $\{0, 1, 2, 3, 4, 5\}$

b $\{x \mid x \in \mathbb{N}, x < 6\}$

2 a True **b** False; $5 \in \mathbb{Q}$ **c** True

d True **e** True

3 a



3 b $\{0,4,6,7,9,10\}$ **c** $\{0,1,2,4,5,7,9,10\}$

d $\{3,8\}$ **e** $\{1,2,3,5,6,8\}$

4 $\left\{\frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0\right\}$

5 a 3 **b** 9 **c** 21

6 a 52 **b** 12

Exercise 1.3

1 a i 764.38 **ii** 764 **iii** 764

b i 234.37 **ii** 234 **iii** 234

c i 0.02 **ii** 0 **iii** 0.0238

d i 0.01 **ii** 0 **iii** 0.00546

e i 15.10 **ii** 15 **iii** 15.1

f i 86.80 **ii** 87 **iii** 86.8

g i 178867.35 **ii** 178867 **iii** 179000

h i 0.58 **ii** 1 **iii** 0.580

i i 29.89 **ii** 30 **iii** 29.9

2 a 0.2615384615, 0.262

b i \$3.92 **ii** \$3.93 Final answer is inaccurate when rounding to 3 s.f. in part **a**

1 $|a+b| \le |a| + |b|$

2 $|a-b| = |b-a|$

3 $|a-b| \ge ||a| - |b||$

4 $|a \times b| = |a| \times |b|$

Exercise 1.4

1 a 15 **b** 6 **c** 30 **d** 30

e 6 **f** 9

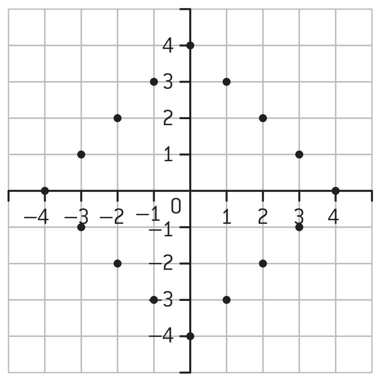
2 a 0.4 **b** 2.5 **c** 0.4 **d** 2.5

e 400 **f** 0.04

3 a 4 **b** $|OA| = |a| + |b|$

c $|-2| + |1| = 3$

d



e Diameter = 8, which is two times the radius

f Measuring distances through all the points along the streets gives a total distance of 32, hence $p = 4$.

Verify the conjecture that p is independent of radius by considering a circle with a different radius.

Exercise 1.5

1 a 5 **b** 3 **c** 10 **d** 3

2 a 6 **b** 34 **c** 1 **d** 9

3 a 12 **b** 9 **c** 2 **d** 1

Investigation 1.2

1 – 6 Circles around 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

4 47 is the largest circled number that has multiples in the square.

7 11, 13, 17, 23, 31, 41, 53, 67, 83, 101, 121

8 All values for $0 \le x \le 9$ are prime. When $x = 10$, 121 is the smallest composite.

9 Does not find 2, 3, 5, 7, 19, 29, 37, 43, 47, 59, 61

Exercise 1.6

1 a prime **b** prime **c** composite

d prime **e** composite

2 a $(a \bullet b) \bullet c = (a+b-2) \bullet c = a+b-2+c-2 = a+b+c-4$

$a \bullet (b \bullet c) = a \bullet (b+c-2) = a+b+c-2-2 = a+b+c-4$

Equal, therefore associative

b $a \bullet e = a \Rightarrow a + e - 2 = a \Rightarrow e = 2$

c i $a \bullet a = 2a - 2$

ii $a \bullet a \bullet a = (2a-2) \bullet a = 3a - 4$

iii $a \bullet a \bullet a \bullet a = (3a-4) \bullet a = 4a - 6$

d $na - 2(n-1)$

e $a \bullet 2 \bullet a \bullet 2 \bullet a \bullet 2 \bullet a \bullet 2 \bullet a \bullet 2 \bullet a = a \bullet a \bullet a \bullet a \bullet a \bullet a = 6a - 10$

Exercise 1.7

1 a 4 **c** 6

b 6 **d** 18

2 a 12

b 40

c 120

d 84

3 a The least common multiple of 4, 6 and 9 is 36

Investigation 1.1

a	b	$\lvert a+b\rvert$	$\lvert a\rvert+\lvert b\rvert$	$\lvert a-b\rvert$	$\lvert b-a\rvert$	$\lvert a\rvert-\lvert b\rvert$	$\lvert\lvert a\rvert-\lvert b\rvert\rvert$	$\lvert a\times b\rvert$	$\lvert a\rvert\times\lvert b\rvert$
2	5	7	7	3	3	−3	3	10	10
−3	4	1	7	7	7	−1	1	12	12
1	−2	1	3	3	3	−1	1	2	2
−5	−4	9	9	1	1	1	1	20	20

b

year	12	24	36	48	60	72	84	96	108	120
no. of populations at their maximum	2	2	3	2	2	3	2	2	3	2

i 0 **ii** 7 **iii** 3

c

year	13	26	39	52	65	78	91	104	117	130
no. of populations at their maximum	0	0	0	1	0	1	0	1	1	0

i 4 **ii** 0 **iii** 0

d It will be best for both populations not to emerge at the same time. If they emerge every 13 and 17 years then they will emerge together every 221 years.

If they emerge every 12 and 18 years they will emerge together every 36 years.

4 **a** **i** $30 = 2\times3\times5 \quad 135 = 3\times3\times3\times5$

ii $30\times135 = 2\times3\times3\times3\times3\times5\times5$

Product of greatest common factor and least common multiple = $2\times3\times3\times3\times3\times5\times5$

Hence both are equal

b Let $n = hd_n$ where d_n is the product of all the factors in n that are not in m.
Similarly let $m = hd_m$

The least common multiple of n and m will therefore be $l = hd_nd_m$
$nm = hd_nhd_m = h(hd_nd_m) = hl$

Exercise 1.8

1 **a** $\frac{3}{7}$ **b** $\frac{3}{8}$ **c** $\frac{1}{3}$ **d** $\frac{1}{8}$

2 **a** $\frac{8}{11}$ **b** $\frac{1}{2}$ **c** $\frac{7}{20}$ **d** $\frac{19}{24}$

3 **a** $\frac{3}{10}$ **b** $\frac{3}{8}$ **c** $\frac{6}{5}=1\frac{1}{5}$ **d** 4

Investigation 1.3

1 $2^5, 2^5, 2^4, 2^7 \quad 2^m\times2^n = 2^{m+n} \quad 3^5, 5^3$ Yes

2 $2^1, 2^2, 2^1, 2^4 \quad 2^m\div2^n = 2^{m-n}$ Yes

3 $2^4, 2^4, 2^6 \quad (2^m)^n = 2^{mn}$ Yes

4 $8^2, 12^2, 10^4 \quad a^m\times b^m = (a\times b)^m$

Exercise 1.9a

1 **a** 32 **b** 27 **c** 1000000 **d** −243 **e** −16

2 **a** 625 **b** 16 **c** 100000 **d** 4096 **e** 27 **f** 18 **g** $\frac{25}{8}$ **h** 49

3 **a** 7.53 **b** 104 **c** 1.41 **d** −0.00243 **e** 39.1

Exercise 1.9b

1 **a** $\frac{1}{9}$ **b** $\frac{1}{5}$ **c** $\frac{1}{64}$ **d** 2 **e** $\frac{9}{4}$

2 **a** $\frac{16}{25}$ **b** 1 **c** 1 **d** $\frac{1}{25}$ **e** $\frac{1}{12}$

3 **a** 3 **b** $\frac{4}{3}$ **c** 32 **d** 8 **e** $\frac{1}{3}$

4 **a** **i** Greatest common factor is 45

ii $3^{45\times3} = (3^3)^{45} = 27^{45}$

$5^{45\times2} = (5^2)^{45} = 25^{45}$

Hence $3^{135} = 27^{45} > 25^{45} = 5^{90}$

b For example $a = -3$ and $n = 2$

5 **a** $1 = k(365.25)^2 \Rightarrow k = 0.00000750$

b $R^3 = 0.00000750\times4333^2 = 140.7...$

$R = 5.20$ AU

c $T = 365.25R^{\frac{3}{2}}$

d $T = 365.25\times(30.07)^{\frac{3}{2}} \approx 60200$ days.

Investigation 1.4

1 All expressions are equal to $2\sqrt{3}$ (3.464…), with the exception of $\frac{12-6\sqrt{3}}{2-\sqrt{3}}$, which is equal to 6.

2 The simplest is $2\sqrt{3}$

3 3.46 $3.46^2 = 11.9716$ $(2\sqrt{3})^2 = 12$ The surd value is always exact.

Exercise 1.10a

1 **a** $4\sqrt{2}$ **b** $2\sqrt{30}$

2 **a** $3\sqrt{2}+2$ **b** $2\sqrt{2}$

Exercise 1.10b

1 **a** $\frac{\sqrt{6}}{3}$ **b** $\sqrt{2}$ **c** $\frac{\sqrt{2}}{2}$

2 **a** $2-\sqrt{3}$ **b** $3(\sqrt{5}+\sqrt{2})$ **c** $\frac{9\sqrt{2}-6}{7}$ **d** $3+2\sqrt{2}$ **e** $\frac{5-2\sqrt{6}}{2}$

Exercise 1.11

1 Saturn furthest, Jupiter largest

2 **a** 3.24×10^8 **b** 4.56×10^5 **c** 1.28×10^{-4} **d** 6.21×10^{-6}

3 **a** 2500 **b** 48.1 **c** 0.0285 **d** 0.000307

4 **a** 1830 **b** 2.67×10^{-26} kg

Chapter 1 test

1 **a** $P = \{4, 5, 6, 7, 8, 9\}$
$Q = \{0, 1, 2, 3, 4, 5, 6, 7\}$

b i F ii F iii T iv T

c $P\cap Q = \{x \mid x\in\mathbb{N}, 4\le x\le7\}$

2 **a** 107 **b** 106.78

3 **a** 0 **b** 0 **c** 128

4 **a** T **b** T **c** F **d** F

5 **a** 3×19 **b** prime **c** prime **d** 11×13 **e** 7×19

6 **a** $\frac{5}{8}$ **b** $\frac{5}{16}$ **c** $\frac{8}{9}$ **d** $\frac{3}{8}$

7 **a** 100 000 **b** 64 **c** $\frac{64}{5}$

8 **a** $3\sqrt{2}$ **b** $4\sqrt{3}$ **c** 6

9 **a** **i** 1.2358×10^8 **ii** 1.27×10^{-3}

b **i** 254 000 **ii** 0.0768

10 **a** $\frac{1.5\times10^p}{2\times10^q} = 0.75\times10^{p-q}$ (M1)(A1)

$= 7.5\times10^{p-q-1}$

i 7.5 **ii** $p-q-1$ A1A1

b $b\times10^6 + c\times10^7 = \frac{b}{10}\times10^7 + c\times10^7$ M1

$= \left(\frac{b}{10}+c\right)\times10^7 = d\times10^7$ (A1)

Hence $d = \frac{b}{10}+c$ (A1)

ii $\frac{b}{10}+c<10$ A1

If not the answer will not be in standard form R1

[Total 9 marks]

11 **a** $t = 0$ gives $5 = a$ A1

$t = 2$ gives $320 = 5\times2^{2b}$ M1

$\Rightarrow 64 = 2^{2b}$

$\Rightarrow 2b = 6$ (M1)

$b = 3$ A1

b $N = 5\times2^9 = 2560$ M1A1

c 8 A1

d Week 4 M1A1

[Total 9 marks]

12 **a** 14 **b** 84

13 **a** 8 **b** 4 **c** 1

14 **a** $2\sqrt{3}$ **b** $\sqrt{2}+1$ **c** $3-\sqrt{6}$

Modelling and investigation

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
$\frac{\text{planet diameter (km)}}{\text{ball diameter (cm)}}$	697	605	637	970	5958	5022	2324	2251

The scales are vastly different, so she cannot use these balls to make an accurate model. For example, the ball used to represent Jupiter should be ten times larger if it was to be a similar scale as that used to represent Mercury.

Sizes would be

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
1	2.5	2.5	1.5	30	25	10	10

Distances from the sun would be

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
10 cm	19 cm	26 cm	39 cm	1.34 m	2.48 m	4.96 m	7.76 m

Using this scale, she has room in the classroom for the model.

Answers – Chapter 2

Exercise 2.1a

1 $10 - 8x$

2 $2a + 5b$

3 $3a + 2a^2$

4 $\frac{4}{x} - \frac{2}{y}$

Exercise 2.1b

1 **a** $3a - 3b$ **b** $2ab + bc$
c $x^2 + 4x$ **d** $xy^2 - x^2y$

2 **a** $2a^2 + 2ab + 2ac$ **b** $ab^2c - bc^2d$

3 **a** $7x + 10$ **b** $a + 6$

Exercise 2.1c

1 **a** $5(x + 3)$ **b** $4(2x - 1)$
c $3(4a + 3)$

2 **a** $2(2p + 3q)$ **b** $b(ac - d)$
c $2x(x - 2)$ **d** $xy(x + 3y^2)$

3 **a** $3ab(2a + 3b)$ **b** $x^2z^2(yz + x^2)$

Exercise 2.2

1 **a** -2 **b** $\frac{2}{3}$ **c** 2

2 550 m

Exercise 2.3a

1 **a** $-\frac{1}{3x}$ **b** $\frac{9 + 8a^2}{12a}$
c $\frac{9a + 2}{12a^2}$ **d** $\frac{a^2 + 2a + 3}{a^3}$

2 **a** $\frac{2x + 3}{(x + 1)(x + 2)}$ **b** $\frac{2x^2 + x - 1}{2x(x - 1)}$
c $\frac{x^2 + 3}{(2x - 3)(x + 2)}$ **d** $\frac{2x^2 + 5x - 2}{x(2x - 1)}$

3 **a** <Answer to come from Peter>

b <answer to come from Peter>

c $m = 2$ in first case and $m = 3$ in second case

d $\frac{1}{m + 1} + \frac{1}{m(m + 1)} = \frac{m + 1}{m(m + 1)} = \frac{1}{m}$.

Exercise 2.3b

1 **a** $2x - 3y$ **b** $\frac{9}{2}x - 9$

2 **a** $\frac{x}{12}(2x + 3)$ **b** $\frac{1}{2}t(2u + at)$

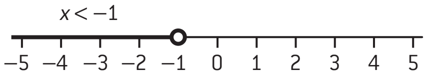
Exercise 2.4

1 $0.6 \leq \frac{11}{17}$

2 3, 4, 5, 6, 7, 8, 9

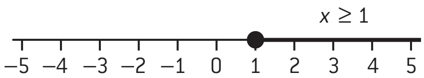
3 **a** F **b** T **c** F **d** F

4 **a**

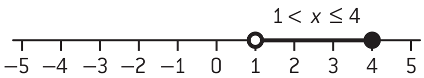


b

c



d



Exercise 2.5a

1 **a** 7 **b** 9 **c** 2 **d** 16

2 **a** 2 **b** 2 **c** 1.5 **d** 3

3 **a** 3 **b** −20 **c** 2 **d** $-\frac{1}{8}$

Exercise 2.5b

1 13

2 5

Exercise 2.5c

1 **a** $x = \frac{1}{2}(y - 1)$

b $x = \frac{y}{4} + 5$

c $x = \frac{1}{2}(5 - y)$

d $x = \frac{3}{y}$

2 $l = \frac{A}{\pi r} - r$

3 **a** $V = IR$

b $R = \frac{V}{I}$

4 $h = \frac{V}{l \times w}$; $h = 0.4$ cm

5 **a** $R_t = \frac{R_1R_2}{R_1 + R_2}$ **b** 0.811

Exercise 2.5d

1 $r = \pm\sqrt{\frac{A}{4\pi}}$

2 $t = \frac{b - a^2}{q}$

3 $b = \sqrt{a^2 - c^2}$

Investigation 2.1

a **i** $4 + 3 \leq 8 + 3$, $7 \leq 11$

ii $4 + (-2) \leq 8 + (-2)$, $2 \leq 6$

iii $4 - 1 \leq 8 - 1$, $3 \leq 7$

iv $4 - (-4) \leq 8 - (-4)$, $8 \leq 12$

v $4 \times 2 \leq 8 \times 2$, $8 \leq 16$

vi $4 \times (-3) \leq 8 \times (-3)$, $-12 \not\leq -24$

vii $4 \div 4 \leq 8 \div 4$, $1 \leq 2$

viii $4 \div (-2) \leq 8 \div (-2)$, $-2 \not\leq -4$

b See key point

c Preserved with addition and subtraction, multiplication changes to equality, and division not possible

d Results are the same (not for multiplication by 0 with $<$ and $>$)

Exercise 2.6

1 **a** $x \leq 7$ **b** $x \leq 4$
c $x \leq \frac{2}{3}$ **d** $x > -6$

Exercise 2.7

1 **a** $x^2 + 4x + 3$
b $x^2 - 2x - 8$
c $x^2 - 3x - 10$
d $x^2 - 8x + 15$

2 **a** $3x(1 - 2x)$ **b** $y(2y + 3)$
c $(2x - 3)(2x + 3)$ **d** $(2 - x)(2 + x)$

3 **a** $(x + 2)(x + 3)$ **b** $(x - 2)(x - 4)$
c $(x + 3)(x - 1)$ **d** $(x + 4)(x - 5)$

4 **a** $9x^2 - 4$ **b** $6x^2 - 5x - 4$

5 **a** $(3x + 2)(x + 2)$ **b** $(x - 3)(2x + 7)$
c $(2x + 5)(x - 3)$ **d** $(3x - 1)(2x + 3)$

Exercise 2.8

1 **a** −5, 0 **b** 0, $\frac{2}{3}$ **c** −3, 3
d −0.4, 0.4 **e** −5, −2 **f** −3, 5
g −3, 2 **h** 3, 7 **i** $-\frac{5}{4}$, 2
j $-\frac{1}{3}$, $\frac{1}{2}$ **k** $\frac{4}{3}$ **l** −7, $\frac{2}{3}$,

2 14, 15 or −15, −14

3 $x \leq -1$ or $x \geq 7$

4 **a** **i** $(x - 1)(x + 5)$ **ii** 1 and -5

b $(x + p)^2 \equiv (x + p)(x + p) \equiv x^2 + px + px + p^2 \equiv x^2 + 2px + p^2$

c $x^2 + 2px + q = 0$
$x^2 + 2px + p^2 = p^2 - q$
$(x + p)^2 = p^2 - q$

d $x = -p \pm \sqrt{p^2 - q}$

e $p = 2, q = -5$ gives

$x = -2 \pm \sqrt{4-(-5)} = -2 \pm 3 = 1$

and -5

Investigation 2.2

2 batteries (x), 3 batteries (y)

y \ x	0	1	2	3	4	5	6	7	8	9	10
0	0	2	4	6	8	10	12	14	16	18	20
1	3	5	7	9	11	13	15	17	19	21	23
2	6	8	10	12	14	16	18	20	22	24	26
3	9	11	13	15	17	19	21	23	25	27	29
4	12	14	16	18	20	22	24	26	28	30	32
5	15	17	19	21	23	25	27	29	31	33	35
6	18	20	22	24	26	28	30	32	34	36	38
7	21	23	25	27	29	31	33	35	37	39	41
8	24	26	28	30	32	34	36	38	40	42	44
9	27	29	31	33	35	37	39	41	43	45	47
10	30	32	34	36	38	40	42	44	46	48	50

2 batteries (x), 3 batteries (y)

y \ x	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

8,0 6,3 4,6 2,9 10,0 9,1 8,2
7,3 6,4 5,5 4,6 3,7 2,8 1,9 0,10

She bought 6 packets of 2 batteries and 4 packets of 3.

Exercise 2.9a

1 **a** $x = 2, y = 1$ **b** $x = 3, y = -2$
c $x = -2, y = 1$ **d** $x = -2\ y = -4$
e $x = 5, y = 1$ **f** $x = 3, y = 7$
g $x = 4, y = 2$ **h** $x = -3, y = 6$
i $x = 7, y = -3$ **j** $x = 1, y = 1$
k $x = 3, y = 2$ **l** $x = 5, y = -1$

Exercise 2.9b

1 **a** $x = 2, y = 5$ **b** $x = 2, y = -2$
c $x = 7, y = 2$ **d** $x = -6, y = 2$

2 **a** Because the second equation can be written as $x + 2y = 5$ which contradicts the first equation.

b $2x + 2ay = 14$
$2x + 3y = b$
$2ay - 3y = 14 - b$
$y(2a-3) = 14 - b$
$y = \dfrac{14-b}{2a-3}$
$x = 7 - \dfrac{a(14-b)}{2a-3}$
$= \dfrac{14a - 21 - 14a + ab}{2a-3} = \dfrac{ab-21}{2a-3}$

c $a = 1.5$, Because for this value of a the first equation can be written as $2x + 3y = 14$, and hence there is no solution if $b \neq 14$ and an infinite number of solutions if $b = 14$

d **i** $b = 14$

ii Any pair of values such that $2x + 3y = 14$ for example $x = 1, y = 4$

3 **a** Maximum value of r would be when $f = 0$ and would be 80

b $r \geq 5f$

c Solve $\begin{matrix} f + 3r = 240 \\ 5f - r = 0 \end{matrix}$

$f = 15, r = 75$

d Any increase in the number of foxes would mean the number of rabbits also has to increase (as $r \geq 5f$), hence $f + 3r \geq 240$, so the population of foxes cannot increase.

An increase of 1 in the number of rabbits would mean the number of foxes would have to reduce by 3 as $f + 3r \leq 240$, so the total population would decrease.

Chapter 2 test

1 **a** 10 **b** 3 **c** 26 **d** $\dfrac{1}{2}$

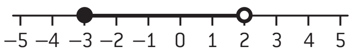
2 **a** $4a - 12$ **b** $2x^2 - 3x$
c $4x + 8$ **d** $4x^2 + 10x + 3$

3 **a** $2(3x + 2)$ **b** $3x(x - 3)$
c $4xy^3(2x + 3yz)$ **d** $\dfrac{x^2}{12}(8x - 9)$

4 $a = \dfrac{1}{2}(P - 2b)$

5 $m = \dfrac{2E}{(v^2 + 2gh)}$

6 $-3 \leq x < 2$



7 **a** 7 **b** 6

8 $x \leq \dfrac{9}{2}$

9 $(x-7)(x+3)$

10 $x = 4, y = -1$

11 **a** $750

b **i** $n = \dfrac{C - 500}{25}$ **ii** $n = \dfrac{400}{25} = 16$

c $\$50n$

d $50n = 500 + 25n \quad n = 20$

12 **a** The number of small squares in the large square is $n \times n = n^2$

The length of the rectangle on top is equal to the figure number and so the number of small squares is n.

The rectangle on the side has a length of one less than the figure number so contains n–1 small squares, Therefore

$N = n^2 + n + n - 1 = n^2 + 2n - 1$

b $N = (n+1)^2 - 2$

c $N = (n+1)^2 - 2 = (n+1)(n+1) - 2$

$= n^2 + 2n + 1 - 2$

$= n^2 + 2n - 1$

13 $x = \pm\sqrt{A^2 - \frac{v^2}{\omega^2}}$

14 $l = \frac{gT^2}{4\pi^2}$

Modelling and investigation

38, 34, 42, $m = 2(n-1)$,

Number of teams	Total number of matches
2	2
3	6
4	12

5 teams 20 matches verified by a list, $n(n-1)$, 90, 380, $n(n-1) = 132$, $n(n-1) = 182$, $n(n-1) = 240$, $n(n-1) = 306$ or $n^2 - n - 132 = 0$, $n^2 - n - 182 = 0$, $n^2 - n - 240 = 0$, $n^2 - n - 306 = 0$, 12, 14, 16, 18.

Answers – Chapter 3

Exercise 3.1

1 **a** 0.015 m **b** 34 g **c** 0.0015 km
d 0.234 g **e** 100 nm **f** 0.0245 kg
g 120 cm **h** 27.5 cm **i** 24 000 μg
j 0.00312 kg **k** 0.023 km
l 430 mg

2 700

3 1/1000

4 0.605

5 50

Exercise 3.2

1 **a** 279 **b** 500 000 **c** 2300
d 0.0343 **e** 0.04 **f** 15 000
g 0.00025 **h** 1220

2 **a** 1250 **b** 0.000456 **c** 1 210 000
d 7.6 **e** 120 **f** 320 000 000
g 0.34 **h** 300 000

3 35

4 1.3 l

5 200

Exercise 3.3a

1 **a** 1:3 **b** 1:1.5 **c** 1:⅓ **d** 1:0.625

2 **a** 2:1 **b** 1.8:1 **c** 0.25:1 **d** 0.4:1

3 5:3, 1:0.6

4 **a** 2 : 3 **b** 3 : 2 **c** 4 : 1
d 16 : 1 **e** 3 : 8 **f** 11 : 40
g 1 : 3 **h** 2 : 3

5 Dean's sister's drink

6 France

7 $1\frac{1}{3}$ tonnes

8 18.8g, 53.8g

Exercise 3.3b

1 \$18 **2** 4 : 1

3 Grass 55m^2 Paving 25m^2

4 Jean 36 Michel 24 Boris 21

5 Gold 4.5g Copper 1.335g
Silver 0.165g

6 310 m **7** 60 cm

Exercise 3.4a

1 **a** 85% **b** 66.7% **c** 51.4%

2 **a** 34% **b** 175% **c** 67.5

3 **i** **a** $\frac{7}{20}$ **b** $\frac{1}{8}$ **c** $\frac{7}{5}$
ii **a** 0.35 **b** 0.125 **c** 1.4

4 **a** 62.5% **b** 42

5 70168

6 360 (the first packet, as only 336 from second packet)

7 **a** Total number of insects is $3000000 \times 300 = 900000000$
Number of insects that can be eaten per day is 9000000
Maximum number of sparrows is $\frac{9000000}{90} = 100000$

b Number of sparrows that can be eaten is $\frac{0.3}{100} \times 100000 = 300$
Maximum population of hawks is 100

c Population of insects reduced by $\frac{4}{5}$ so sustainable sparrow population also reduced by $\frac{4}{5}$ to 20000

d Number of sparrows being eaten per day is 300
Number of days until extinct is $\frac{20000}{300} \approx 67$ days

e The population of insects will quickly increase and the population of hawks will disappear.

Exercise 3.4b

1 \$129.60

2 \$37134

3 \$119

4 \$8.40

5 **a** \$2.59 **b** \$22

6 21% increase

Exercise 3.5

1 £48 **2** \$225 **3** 4 km

4 \$1.40 **5** \$9.50

6 **a** Sugar $\frac{0.75 \times 160}{1000} = 0.12$

Cinnamon $\frac{0.80 \times 5}{40} = 0.1$

Cooking apples $\frac{1.85 \times 600}{1000} = 1.11$

Plain flour $\frac{1.80 \times 400}{1000} = 0.72$

Butter $\frac{1.50 \times 200}{250} = 1.2$

Eggs $\frac{1.80}{12} = 0.15$

Total cost €3.40

b $\frac{3.40}{6 \times 4.50} \times 100 = 12.6\%$

7 **a** $x = \frac{5 \times 15}{12} = 6.25$

b $\frac{y}{9} = \frac{12}{5} \Rightarrow y = \frac{12 \times 9}{5} = 21.6$

c $\frac{a}{6} = \frac{2}{a+1} \Rightarrow a(a+1) = 12$
$\Rightarrow a^2 + a - 12 = 0$
$\Rightarrow (a+4)(a-3) = 0$ $a = -4$ or 3

8 Let the number of fish in the lake be x. Assuming the proportion of fish in the sample is the same as the proportion of fish in the lake we have
$\frac{60}{x} = \frac{12}{80} \Rightarrow x = \frac{60 \times 80}{12} = 400$

Investigation 3.1

0.64, 0.36, 1.41, 0.88, 1.19, 1.75 USD. IN, CH, RU, SW, JA, US.

31.41, 31.29, 59.40, 69.53, 109.88, 44.59 USD. IN, CH, US, JA, RU, SW. Reasons involving the cost of locally produced vs imported goods.

Exercise 3.6

1 **a** 26.67 **b** 51.71 **c** 4760.53
d 21.24 **e** 81.82 **f** 6.56 **g** 2442
h 1.61

2 **a** 6537.50, **b** 1500, **c** 268.82,

3 \$1 : € 1.10

Chapter 3 test

1 **a** 2340 m **b** 3.25 m
c 0.00025 mm **d** 0.125 kg

2 21.8 t

3 **a** 3.88 cm^2 **b** 0.494 g

4 2.5 cm

5 **a** 17.2 cm^2 **b** 3450 mm^3
c 25 cm^2 **d** 0.0236 m^2

6 200 times larger

7 **a** 5 : 8 **b** 1 : 1.6

8 **a** 7 : 3 = 2.33 : 1

b *x* /*x* + 44 = 3/7, 110 cakes

9 **a** 35 cm

10 9250 IDR; 125400 IDR

11 80 ml

Modelling and investigation

	11	12	13	14	16	18	20	22	25	28	32
53	4.82	4.42	4.08	3.79	3.31	2.94	2.65	2.41	2.12	1.89	1.66
39	3.55	3.25	3.00	2.79	2.44	2.17	1.95	1.77	1.56	1.39	1.22

4.82, 4.42, 4.08, 3.79, ~~3.55~~, 3.31, ~~3.25~~, ~~3.00~~, 2.94, ~~2.79~~, 2.65, 2.44, ~~2.41~~, 2.17, ~~2.12~~, 1.95, ~~1.89~~, 1.77, ~~1.66~~, 1.56, 1.39, 1.22

	10	11	12	13	14	16	18	20	22	25	28
48	4.80	4.36	4.00	3.69	3.43	3.00	2.67	2.40	2.18	1.92	1.71
34	3.40	3.09	2.83	2.62	2.43	2.13	1.89	1.70	1.55	1.36	1.21

4.80, 4.36, 4.00, 3.69, 3.43, ~~3.40~~, ~~3.09~~, 3.00, ~~2.83~~, 2.67, ~~2.62~~, 2.43, ~~2.40~~, ~~2.18~~, 2.13, ~~1.92~~, 1.89, ~~1.71~~, 1.70, 1.55, 1.36, 1.21

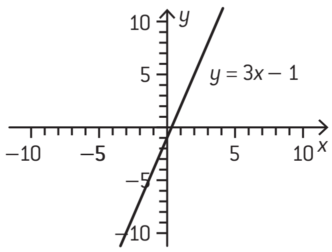
The ratios are very similar.

Exercise 4.1

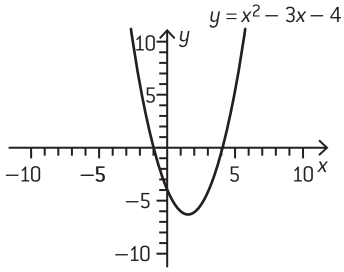
1 **a** function many-to-one

b not a function

2

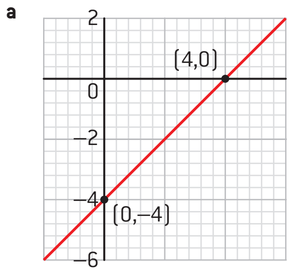


3

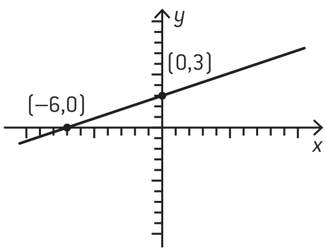


Exercise 4.2

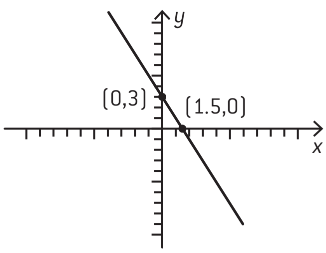
1 **a**



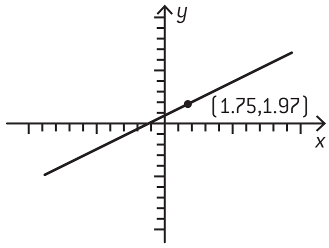
b



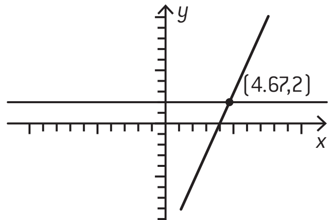
c



2

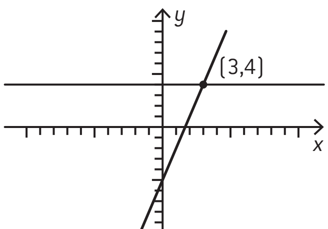


c



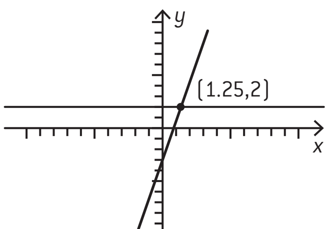
$x = 4.67$

3 **a**



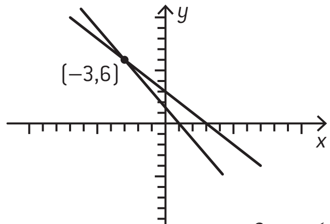
$x = 3$

b



$x = 1.25$

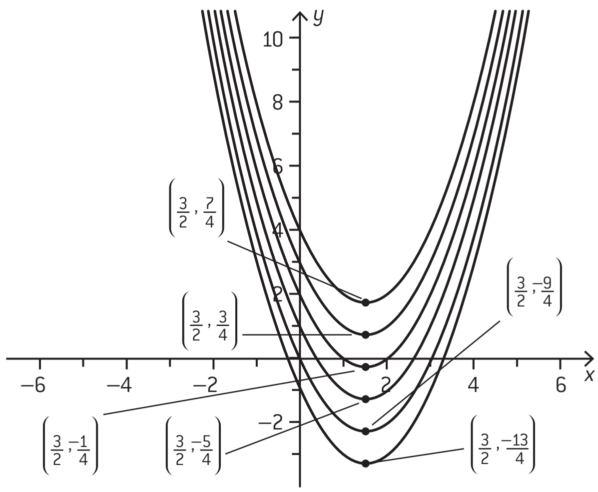
4



$x = -3, y = 6$

Investigation 4.1

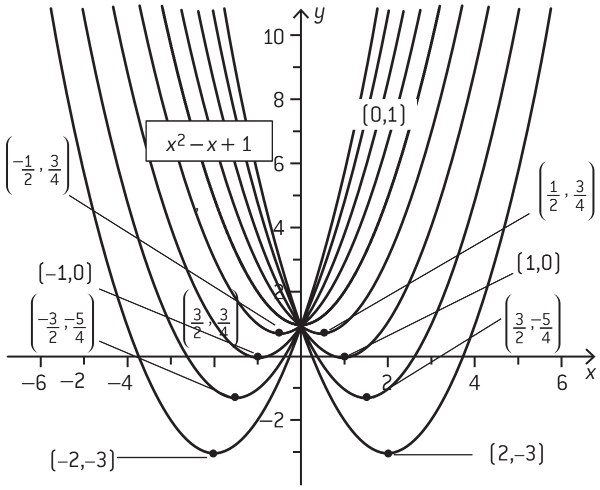
1



a y-intercepts $= c$, value of y when $x = 0$.

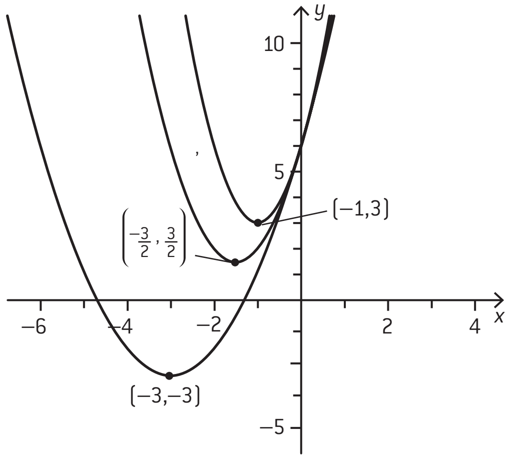
b Vertex is $\left(\frac{3}{4}, -\frac{9}{4} + c\right)$

2



$x = -\frac{b}{2}$, (−3, −8)

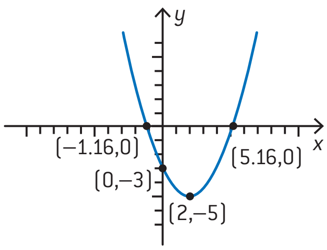
3



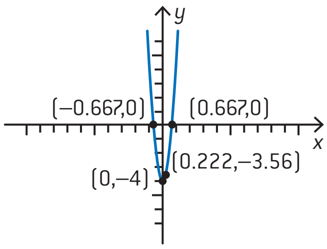
$x = -\frac{b}{2a}$, −0.6

Exercise 4.3

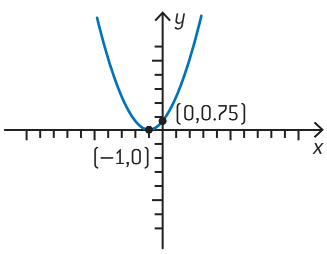
1 a



b



c



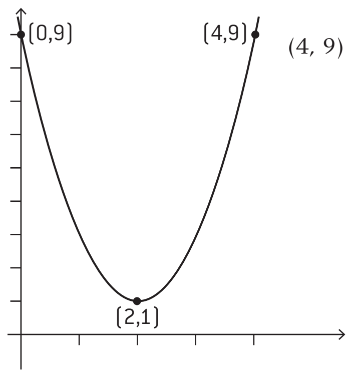
2 (1.5, 4.75)

3 a −3, 3 b −3, 1 c −3

4 (2.5, −2.25)

5 a 9

b



(4, 9)

c $4a + 2b + 9 = 1$ $4a + 2b = -8$

$16a + 4b + 9 = 9$ $4a + b = 0$

$b = -8,\ a = 2$

d Equation of the curve is

$y = 2x^2 - 8x + 9$

Investigation 4.2

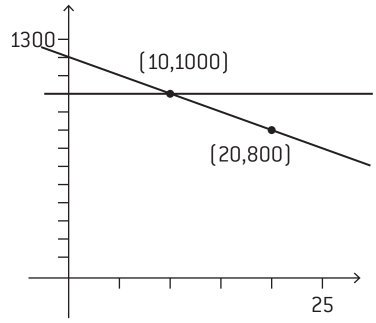
y-intercepts: −4, −3, −2, −1, 0, 1, 2, 3, 4. The y-intercepts are the constant terms in the equations.

gradients: −4, −3, −2, −1, 0, 1, 2, 3, 4. The gradients are the coefficients of x.

y-intercepts: −3, −2, −1, 0, 1, 2, 3. The y-intercepts are the constant terms in the equations.

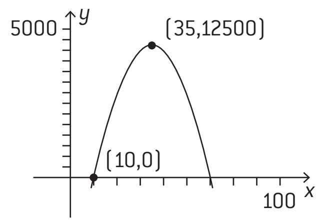
Exercise 4.4

1



a 800 b 10

2



a 10 b 12500, 35

3 a $a = \frac{3.2 - 1.2}{5} = 0.4$ $b = 1.2$

b $C = 0.4 \times 12 + 1.2 = 6.0$ M

c Maximum rate of change of reaction is $a = \frac{3.25 - 1.15}{5}$

$= \frac{2.1}{5} = 0.42$

$C = 0.42 \times 12 + 1.15 = 6.19$

Minimum value of gradient

$a = \frac{3.15 - 1.25}{5} = \frac{1.9}{5} = 0.38$

$C = 0.38 \times 12 + 1.25 = 5.81$

largest percentage error is

$\frac{0.19}{5.81} \times 100 = 3.3\%$

d Maximum rate of change of reaction is

$a = \frac{3.25 - 1.15}{5} = \frac{2.1}{5} = 0.42$

$C = 0.42 \times 12 + 1.15 = 6.19$

Minimum value of gradient

$a = \frac{3.15 - 1.25}{5} = \frac{1.9}{5} = 0.38$

$C = 0.38 \times 12 + 1.25 = 5.81$

4 a i $m = \frac{40 - 100}{4.00 - 2.50} = -40$

$d = -40p + c$ substitute one of the points to get $d = -40p + 200$

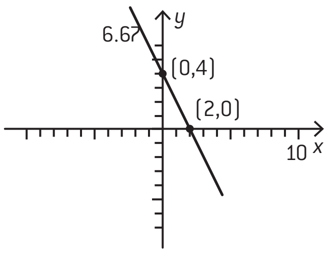
ii $0 = -40p + 200 \Rightarrow p = 5$ so £5

b $R = p(-40p + 200) = 200p - 40p^2$

c Maximum value of R occurs when $p = 2.50$ $R = 250$ so maximum revenue is £250.

Chapter 4 test

1



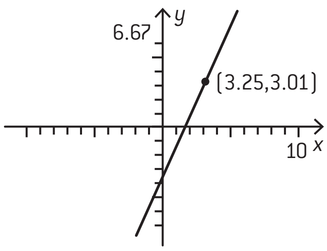
2 marks for graph

x-intercept (2, 0) 1 mark

y-intercept (0, 4) 1 mark

[4 marks]

2

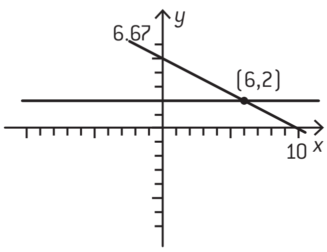


2 marks for graph

$y = 3.01$ 1 mark

[3 marks]

3

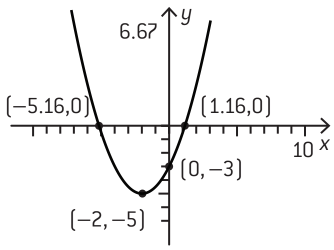


2 marks for graph

$x = 6$ 1 mark

[3 marks]

4



2 marks for graph

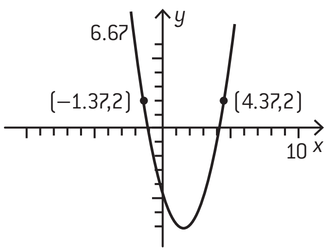
x-intercepts (−5.16, 0) and (1.16, 0) 2 marks

y-intercept (0, −3) 1 mark

vertex at (−2, 5) 1 mark

[6 marks]

5

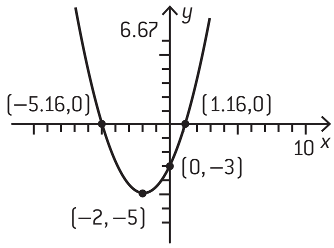


2 marks for graph

$x = -1.37$ and 4.37 2 marks

[4 marks]

6



2 marks for graph

x-intercepts (−4.24, 0) and (0.236, 0) 2 marks

y-intercept is (0, −1)

vertex (−2, 5) 1 mark

$-2-(-4.24) = 2.24$ and $0.236-(-2) = 2.24$ 1 mark

[6 marks]

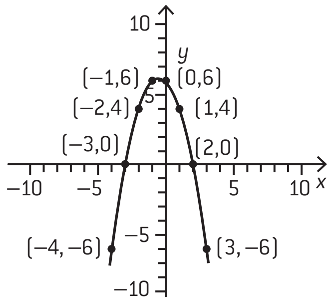
7 **a** one-to-many, not a function 1 mark

b one-to-many, not a function 1 mark

[2 marks]

8 (−4, −6), (−3, 0), (−2, 4), (−1, 6), (0, 6), (1, 4), (2, 0), (3, −6)

4 marks

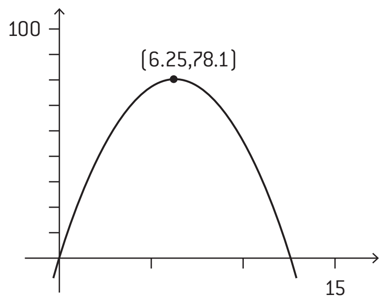


3 marks for graph

[7 marks]

9 $b = 25 - 2x$ 1 mark

$A = x(25 - 2x)$ 1 mark



2 marks for graph

Maximum area is 78.1 m². 1 mark

[5 marks]

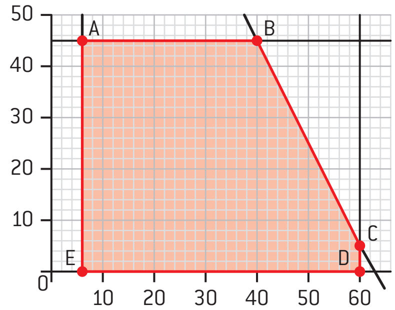
Modelling and investigation

1 $1000x + 500y \leq 62500$, divide by 500 to get $2x + y \leq 125$

2 **a** $x \leq 60$, $y \leq 45$

b $x \geq 4$, $y \geq 0$

3



4 $A = x + 2y$
Maximum will occur at either points C or B as the value of A at all other points will be less than at one of these two points.

B: $x + 2y = 40 + 2 \times 45 = 130$ tonnes

C: $x + 2y = 60 + 2 \times 5 = 70$ tonnes

Hence optimal solution is 40 trips by plane and 45 by truck which will deliver 130 tonnes.

Answers – Chapter 5

Investigation 5.1

1 a and b add up to 180, b and c add up to 180°, a and c are equal, b and d are equal.

2 $b = 180 - a$ since angles on a straight line add up to 180°; $c = a$ since opposite angles are equal; $d = 180 - a$ since angles on a straight line add up to 180°.

3 a and e are equal.

4 c and h add up to 180°.

Exercise 5.1

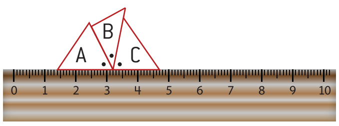
1 **i** $a = 63°$ **ii** $b = 90°$ $c = 64°$
iii $d = 43°$ **iv** $e = 153°$ **v** $f = 22°$
vi $g = 40°$

2 **i** $a = 41°$ **ii** $b = 143°$ **iii** $c = 38°$
iv $d = 70°$ **v** $e = 66°$ $f = 114°$
vi $g = 62°$ $h = 118°$

3 $p = 47°$, $q = 47°$, $r = 133°$, $s = 47°$, $t = 103°$, $u = 30°$, $v = 133°$; yes parallel: t and 103° are alternate angles

Investigation 5.2

a



Angles fit on a straight line, their sum is 180°

b **i** ABC; equal (alternate angles)

ii ACB; equal (alternate angles)

iii sum is 180° (angles on a straight line)

iv angles of a triangle sum to 180° (same result)

Exercise 5.2

1 $a = 64°$,

2 $b = 36°$, $c = 65°$,

3 $d = 77°$, $e = 97°$, $f = 29°$,

4 $g = 45°$, $h = 135°$,

5 $m = 80°$, $n = 60°$,

6 $p = 41°$, $q = 43°$

7 $p + q + r = 180°$, $r + s = 180°$, $p + q = s$

Exercise 5.3a

1 **i** $a = 46°$ **ii** $b = 55°$ **iii** $c = 37°$

2 **i** $a = 120°$ **ii** $b = 71°$ $c = 33°$
iii $d = 60°$ $e = 30°$

Investigation 5.3

	parallelogram	rhombus	rectangle	square	kite	trapezoid
equal	no	no	yes	yes	no	yes
perpendicular	no	yes	no	yes	yes	no
bisected	yes	yes	yes	yes	one	no

Exercise 5.3b

1 **i** c, h, l, m, n,

ii b, c, d, h, i, l, m, n

iii a, c, e, f, h, l, m, n

iv a, b, c, d, e, f, h, i, l, m, n

v d, h, i, j, k

vi g

2 **a** $a = 45°$ $b = 72°$ $c = 63°$
b $d = 104°$ **c** $e = 67°$
d $f = 30°$ $g = 135°$

3 **a** $h = 54$ $i = 117$ **b** $j = 45$

4 **a** $p = 51,\ q = 70$ **b** $r = 76$

5 **a** $\alpha = \frac{360}{6} = 60°$

b 120°

c **i** At the point where the 3 hexagons meet the interior angles add up to $3 \times 120 = 360°$ hence there will be no gaps and the wall can be completely covered.

ii The interior angle must be a factor of 360°

iii The regular polygons are the equilateral triangle, the square and the pentagon.

The triangle and square will completely cover the wall as their interior angles are 60° and 90° which are both factors of 360°.

The interior angle of a pentagon can be found using the same method as used in part b and is 108°, this is not a factor of 360° so the pentagon will not completely fill the wall.

iv The hexagon fills the space with 3 hexagons at each point.

Regular polygons with more sides than a hexagon will have a larger interior angle than 120°

The next largest factor of 360° is 180° (2×180).

It is not possible for a polygon to have an interior angle of 180°.

d One of the regular polygons is a square

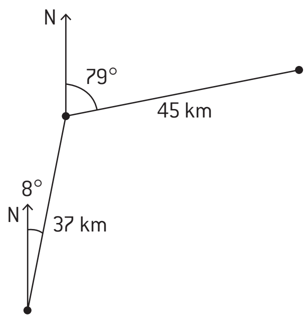
The interior angle of the other polygon is $\frac{360 - 90}{2} = 135°$

Reversing the process of part b (or otherwise) the polygon must have 8 sides.

Exercise 5.4

1 **A** 027°, **B** 064°, **C** 131°, **D** 192° **E** 270°

2



3 270°, 030°, 150°

4 317°

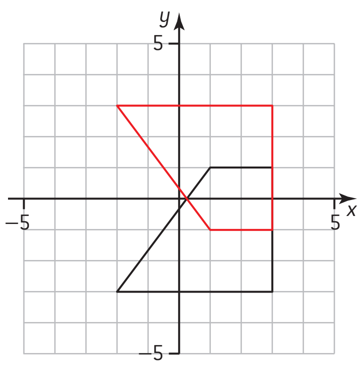
5 147°

6 **i** $a = 92°$ **ii** $b = 266°$

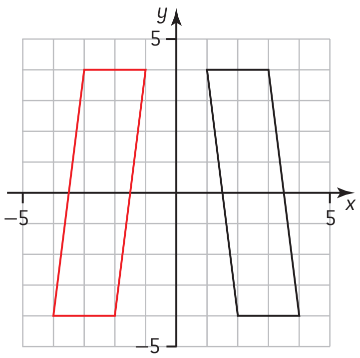
Exercise 5.5a

1 **a** $y = 0$ **b** $x = 0$ **c** $y = x$
d $y = -x$

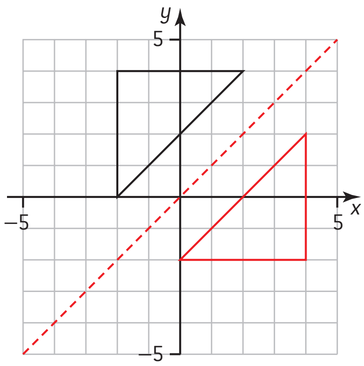
2 **a**



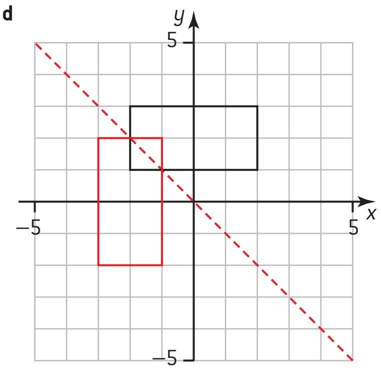
b



c



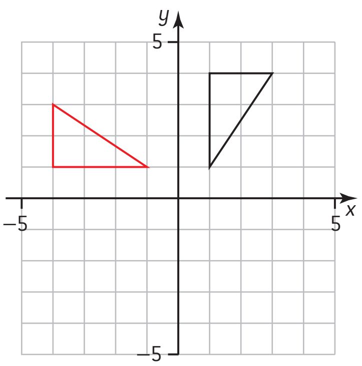
d



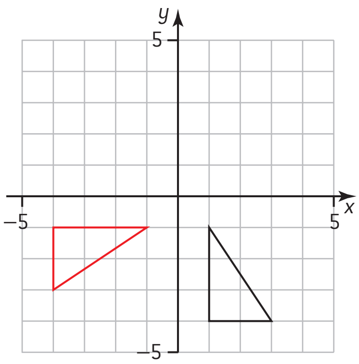
Exercise 5.5b

1 **a** (3, −2) **b** (2, −3) **c** (4, −1)

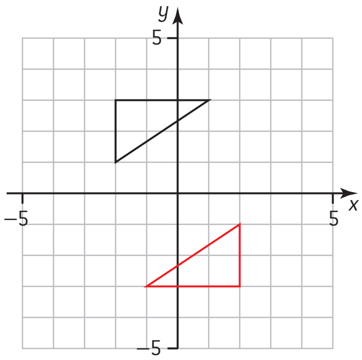
2 **a**



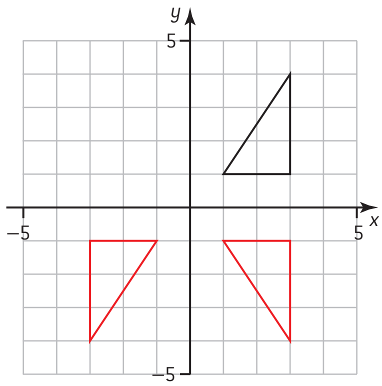
b



c



3



180° rotation about (0, 0)

4 **a** **i** $(-a, b)$

ii (b, a)

iii $(-b, -a)$

b Change the sign on the x-coordinate brings if back to (a, b)

c **i** Swap over the x and y coordinates

ii Swap over the x and y coordinates and change their signs

d First transformation takes to (b, a) and then swap x and y and change the signs to get $(-a, -b)$

e $(-b, a)$, change the sign of the y coordinate and swap the x and y coordinates.

f **i** $(-a, -b)$

ii $(b, -a)$

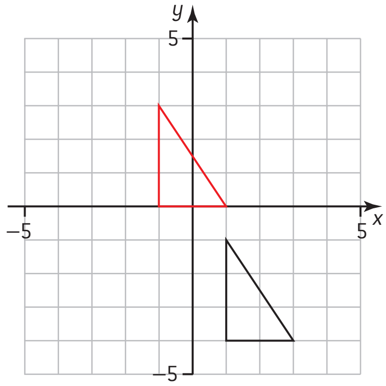
iii (a, b)

g **i** Four rotations of 90° returns the point to its original position

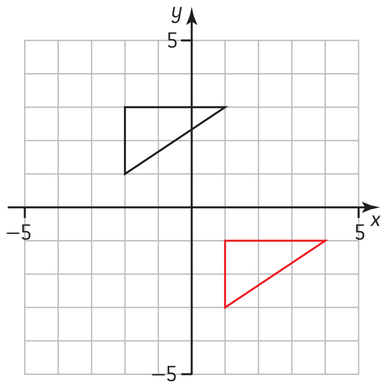
ii A rotation of 180° about (0,0)

Exercise 5.5c

1



2



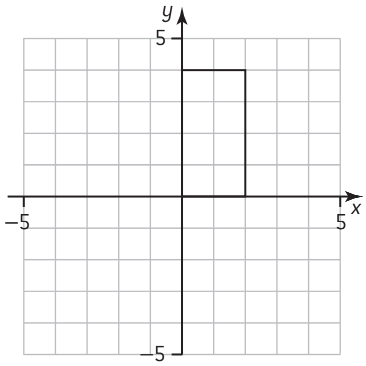
3 5 units to the right and 1 unit up

4 **a** Oppositely congruent

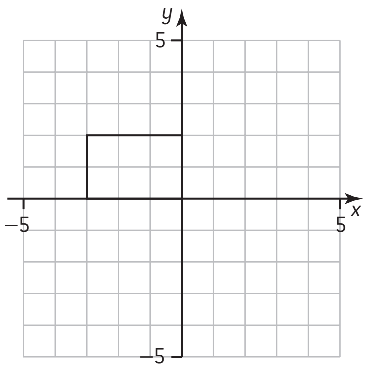
b Directly congruent

c Directly congruent

5 **a** Draw a graph with image of a rectangle with corners



b Draw a graph with image of a rectangle with corners



6 **a** $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

c $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ **d** $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

Investigation 5.4

a pentagons, triangles, parallelograms

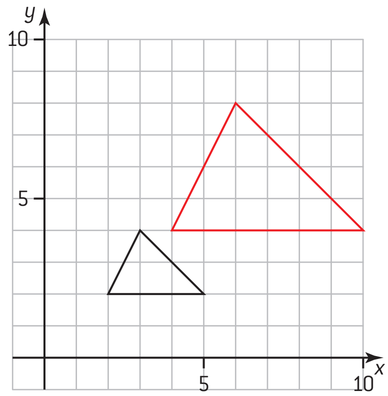
b 1:2, 1:3, 1:1.5; all the sides of each figure are in the same ratio

c The angles of each figure are equal

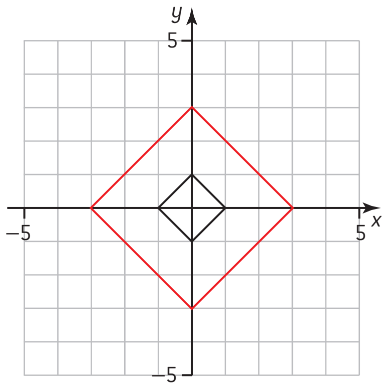
Yes, the objects and the images are similar.

Exercise 5.5d

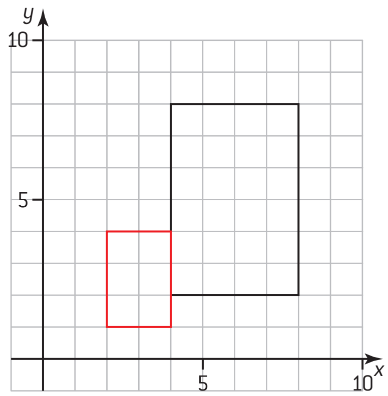
1



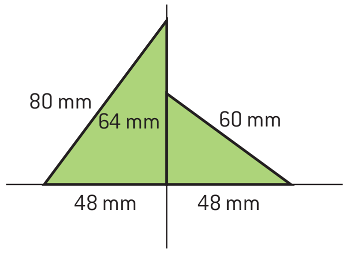
2



3

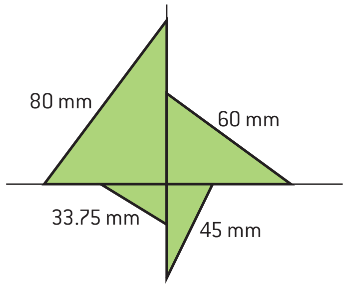


4 **a**



b 60 mm, 48 mm, 36 mm

c

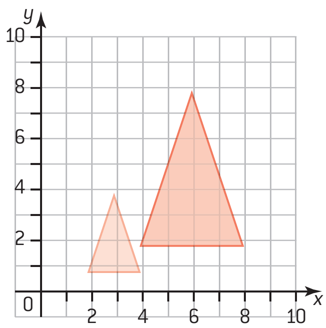


d 45 mm, 36 mm, 27 mm
33.75 mm, 27 mm, 20.25 mm

e $(80+60+45+33.75)+(64-36)$
$+(48-27)+(36-20.25)$
$+(48-27) = 304.5$ mm

5 **a** Area $= \frac{1}{2} \times 2 \times 3 = 3$

b **i**



ii 12

c Length of base is pb and height is ph

Area is

$\frac{1}{2}(pb)(ph) = p^2 \times \frac{1}{2}bh = p^2 A$

Chapter 5 test

1 $75 + a = 180$
$a = 180 - 75$
$a = 105°$

1 mark for equation 1 for solving
1 for solution

[3 marks]

2 $29 + 151 + 57 + b + b = 360$
$237 + 2b = 360$
$2b = 123$
$b = 61.5°$

1 mark for equation 2 for solving
1 for solution

[4 marks]

3 $40 + c + 62 + 28 = 180$
$c + 130 = 180$
$c = 50°$

1 mark for equation 1 for solving
1 for solution

[3 marks]

4 $d = 68°$ 1 mark
$e + 68 = 180$
$e = 112°$

1 mark for equation 1 for solution

[3 marks]

5 $f = 57°$ 1 mark
$g = 180 - 57$ 1 mark for equation
$g = 123°$ 1 for solution

$h + 95 + 57 = 180$
$h + 152 = 180$
$h = 28°$

1 mark for equation 1 for solving
1 for solution

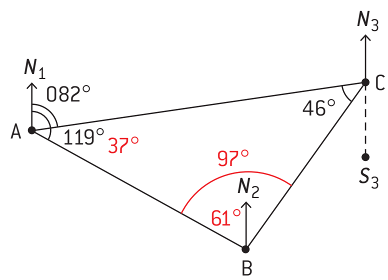
[6 marks]

6 $43 + m = 65$
$m = 22°$

1 mark for equation 1 for solution

[2 marks]

7



$A\hat{B}N_2 = 180 - 119$ 2 marks
$= 61°$

$B\hat{A}C = 119 - 82$ 2 marks
$= 37°$

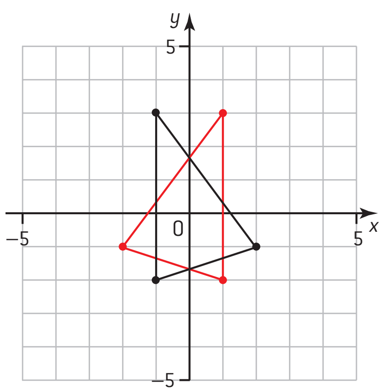
$A\hat{B}C = 180 - (37 + 46)$ 2 marks
$= 97°$

$C\hat{B}N_2 = 97 - 61$ 2 marks
$= 36°$

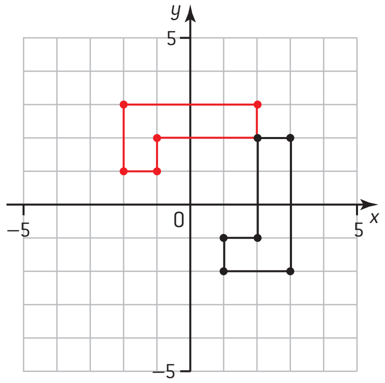
The bearing of C from B is 036° 1 mark

[9 marks]

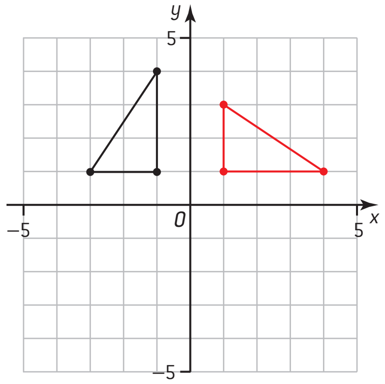
8



9



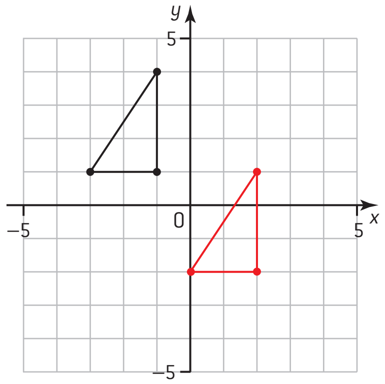
10



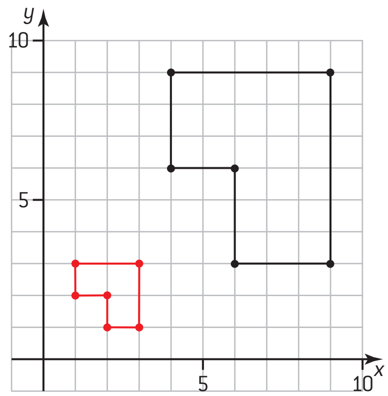
1 mark for each point

[3 marks]

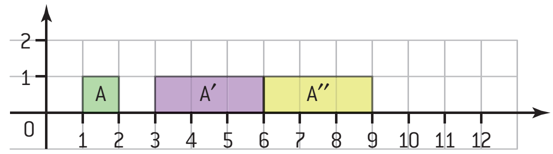
11



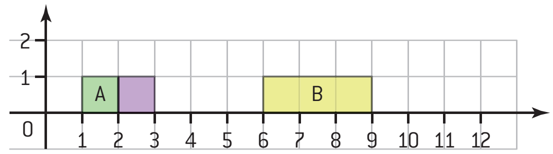
12



13 a and **b**



c $d = 1$



14 a $x = 25°$

b Other proofs are possible

$E\hat{H}D = 180 - a$
Co-interior angles

$G\hat{H}F = 180 - a$
Vertically opposite angles

$H\hat{G}F = b$
Vertically opposite angles

$x = 180 - (180 - a) - b = a - b$

Angles in a triangle sum to 180°

Modelling and investigation

1 a AD = CD
b three sides are equal
c congruent (SSS)
d equal
e equal

2 a equal **b** equal **c** equal (vertically opposite)
d congruent (SAS)

3 a equal (alternate angles)
b equal (alternate angles)
c congruent (AAS) **d** equal

4 a AM = BM, PM is common, right-angle at *P* so congruent (RHS) **b** BM = CM, QM is common, right-angle at *Q* so congruent **c** AP = BP and BQ = CQ **d** circle will pass through A, B and C

Investigation 6.1

1 90°

2 square of side c

3 GQ = a, SE = a

4 $a + a - \text{SP} = a + b$, $\text{SP} = a - b$

5 $4 \times \frac{1}{2}ab + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$

6 c^2, $a^2 + b^2 = c^2$

Exercise 6.1

1 a 1.75 cm
  b 5.97 cm
  c 6.87 cm

2 a 3.77 cm
  b 4.16 cm
  c 55.3 mm

3 5.20 cm

4 $x = 4.5$; lengths of sides are 6, 4.5 and 7.5

5 251 km

6 3.56 m

7 $\text{AC}^2 = \text{AB}^2 + \text{BC}^2$

8 4 cm, $3^2 + 4^2 = 5^2$

9 4.47 cm

10 124 cm

11 9 cm

Investigation 6.2

1 (10, 3)

2 a (6, 3) b (10, 5)

3 (6, 5)

4 $(\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2))$

5 (10, 3)

6 AR = 8, BR = 4

7 8.94

8 $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Exercise 6.2

1 (1, ½)

2 a (3, 4) b (½, 7) c (½, −3) d (½, 2½)

3 8.06

4 a 1.41 b 5 c 11.7 d 9.22

5 a Because any deviation from the line which makes an angle of 90° will form a hypoteneuse of a triangle which will be longer than the side which is at 90°

b Using the point (0,5) distance = $\sqrt{5^2 + 30^2} = \sqrt{925}$

c $q = 2p + 5$

d Distance between (0,5) and $(p, 2p+5)$ is $\sqrt{p^2 + (2p)^2} = \sqrt{5p^2}$

Distance between (30,0) and $(p, 2p+5)$ is $\sqrt{(30-p)^2 + (2p+5)^2}$

$= \sqrt{900 - 60p + p^2 + 4p^2 + 20p + 25}$

$= \sqrt{925 + 5p^2 - 40p}$

Hence $5p^2 + 925 + 5p^2 - 40p = 925$

$10p^2 - 40p = 0 \Rightarrow p = 0$ or 4.

Reject $p = 0$ as it gives (0,5) so just gives the original line.

Hence answer is (4,13)

Exercise 6.3a

1 a 0.719 cm b 0.934 cm c 19.1 cm

2 a 4.72 cm b 5.27 cm c 3.21 cm d 8.15 cm e 9.82 cm f 11.2 cm

3 a 5.95 cm b 8.6 cm c 6.33 cm d 51.8 cm

4 a 4.81 cm b 3.71 cm c 3.38 cm d 2.48 cm e 6 cm f 8.17 cm

5 599 km

6 38.9 m

Exercise 6.3b

1 a 52.5° b 79.4° c 41.7°

2 a 35.5° b 59.6° c 37.1° d 41.1° e 39.7° f 35.7°

3 219.2

4 47.5°

Exercise 6.3c

1 28.6°

2 65.8 m

3 a 7.79 km b 056.5° c 1.84 km d 123.1° e 249.6°

Chapter 6 test

1 a $a^2 = 2.0^2 + 4.8^2$
$a = \sqrt{2.0^2 + 4.8^2}$
$a = 5.2$ cm

1 substitution 1 for solving 1 for solution

b $b^2 = 6.5^2 + 7.8^2$
$b = \sqrt{6.5^2 + 7.8^2}$
$b = 10.2$ cm

1 substitution 1 for solving 1 for solution

c $c^2 = 3.95^2 - 2.43^2$
$c = \sqrt{3.95^2 - 2.43^2}$
$c = 3.11$ cm

1 substitution 1 for solving 1 for solution

d $d^2 = 18.9^2 - 5.7^2$
$d = \sqrt{18.9^2 - 5.7^2}$
$d = 18.0$ cm

1 substitution 1 for solving 1 for solution

e PR = PQ + QR = 1.5 + 2.3 = 3.8 cm   1 mark

$\text{PS}^2 = 4.6^2 - 3.8^2$
$\text{PS} = \sqrt{4.6^2 - 3.8^2}$
$\text{PS} = 2.59$ cm

1 substitution 1 for solving 1 for solution

$e^2 = 1.5^2 + 2.59^2$
$e = \sqrt{1.5^2 + 2.59^2}$
$e = 2.99$ cm

1 substitution 1 for solving 1 for solution

[19 marks]

2 51.1 km

3 $p^2 = 4.1^2 - 3.2^2$
$p = \sqrt{4.1^2 - 3.2^2}$
$p = 2.56$ cm

1 substitution 1 for solving 1 for solution

$q^2 = 2.56^2 + 4.5^2$
$q = \sqrt{2.56^2 + 4.5^2}$
$q = 5.18$ cm

1 substitution 1 for solving 1 for solution

$r^2 = 4.5^2 + 4.3^2$
$r = \sqrt{4.5^2 + 4.3^2}$
$r = 6.22$ cm

1 substitution 1 for solving 1 for solution

$s^2 = 4.3^2 + 3.2^2$
$s = \sqrt{4.3^2 + 3.2^2}$
$s = 5.36$ cm

1 substitution 1 for solving 1 for solution

[12 marks]

4 midpoint $= \left(\frac{5+1}{2}, \frac{8+2}{2}\right)$
$= (3,5)$

2 substitution 2 for solution

[4 marks]

5 distance $= \sqrt{(4+3)^2 + (-2-6)^2}$
$= \sqrt{7^2 + 8^2}$
$= \sqrt{113}$
$= 10.6$

2 substitution 1 for solution

[3 marks]

6 **a** 0.559 1 mark
b 0.616 1 mark
c 0.231 1 mark
d 13.6° 1 mark
e 39.2° 1 mark
f 33.1° 1 mark

[6 marks]

7 **a**
$$\frac{a}{5.67} = \sin 37°$$
$$a = 5.67 \sin 37°$$
$$a = 3.41 \text{ cm}$$

1 substitution 1 rearranging
1 solution

b
$$\frac{b}{6.6} = \tan 49°$$
$$b = 6.6 \tan 49°$$
$$b = 7.59 \text{ cm}$$

1 substitution 1 rearranging
1 solution

c
$$\frac{c}{7.67} = \cos 53°$$
$$c = 7.67 \cos 53°$$
$$c = 4.62 \text{ cm}$$

1 substitution 1 rearranging
1 solution

d
$$\frac{1.95}{d} = \cos 55°$$
$$d \cos 55° = 1.95$$
$$d = \frac{1.95}{\cos 55°}$$
$$d = 3.40 \text{ cm}$$

1 substitution 2 rearranging
1 solution

e
$$\frac{12.1}{e} = \tan 33°$$
$$e \tan 33° = 12.1$$
$$e = \frac{12.1}{\tan 33°}$$
$$e = 18.6 \text{ cm}$$

1 substitution 2 rearranging
1 solution

f
$$\frac{7.89}{f} = \sin 48°$$
$$f \sin 48° = 7.89$$
$$f = \frac{7.89}{\sin 48°}$$
$$f = 10.6 \text{ cm}$$

1 substitution 2 rearranging
1 solution

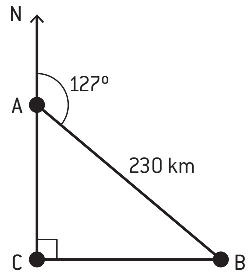
[21 marks]

8 3.20 m

9 **a** 51.5° **b** 46.7° **c** 26.7°

10 056.3°

11 Draw a diagram



1 mark

Angle $B\hat{A}C = 180 - 127 = 53°$

1 mark

Distance east is CB.

$$\frac{CB}{230} = \sin 53°$$
$$CB = 230 \sin 53°$$
$$CB = 184 \text{ km}$$

1 substitution 1 rearranging
1 solution

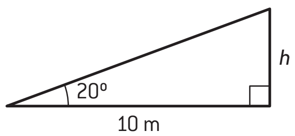
Distance south is AC

$$\frac{AC}{230} = \cos 53°$$
$$AC = 230 \cos 53°$$
$$AC = 138 \text{ km}$$

1 substitution 1 rearranging
1 solution

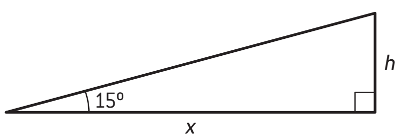
[8 marks]

12 Calculate h



$$\frac{h}{10} = \tan 20°$$
$$h = 10 \tan 20°$$
$$h = 3.64$$

1 substitution 1 rearranging
1 solution



1 new diagram

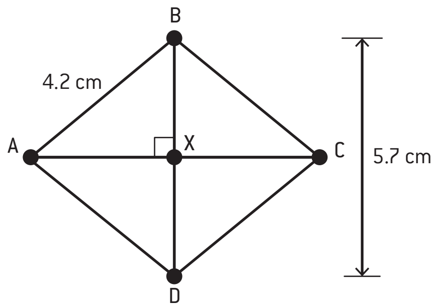
$$\frac{h}{x} = \tan 15°$$
$$x \tan 15° = h$$
$$x = \frac{h}{\tan 15°}$$
$$x = 13.58 \text{ m}$$

1 substitution 1 rearranging
1 solution

The further distance is
13.58 − 10 = 3.58 m 1 mark

[8 marks]

13 Draw a diagram



If BD = 5.7 then $BX = \frac{1}{2}BD = 2.85$

2 mark

$$\sin B\hat{A}X = \frac{2.85}{4.2}$$
$$B\hat{A}X = \sin^{-1}\left(\frac{2.85}{4.2}\right)$$
$$B\hat{A}X = 42.7°$$

1 substitution 1 rearranging
1 solution

Hence $B\hat{A}D = 2 \times 42.7 = 85.5°$

2 marks

and $A\hat{D}C = 180 - 85.5 = 94.5°$

2 marks

[9 marks]

14

a $\sqrt{2}$

b $\sqrt{8-2} = \sqrt{6}$

c i $c^2 + c^2 = 6 \Rightarrow c = \sqrt{3}$

ii $d^2 + d^2 = 2 \Rightarrow d = 1$

e Two sides of the top triangle have lengths $\sqrt{3} - 1$ and $\sqrt{3} + 1$

i $\frac{\sqrt{3}+1}{2\sqrt{2}} \left(\text{or } \frac{\sqrt{6}+\sqrt{2}}{4}\right)$

ii $\frac{\sqrt{3}+1}{\sqrt{3}-1} \left(\text{or } 2+\sqrt{3}\right)$

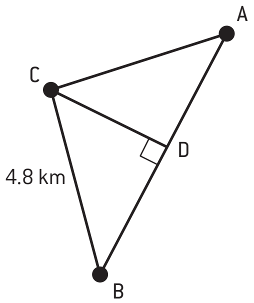
15 **a** Angles in the triangle 50° and 15°

Height of the triangle = $2 \sin 50° \approx 1.53$

Distance = $2 \cos 50° + \frac{1.53}{\tan 15°}$
$= 7.00$ km

b **i** 45°

ii



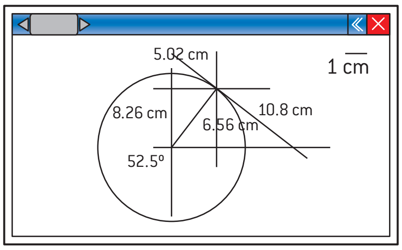
$BD = CD = 4.8\cos 45° \approx 3.394$

$AD = 6.2 - 3.394 = 2.806$

$AC = \sqrt{3.394^2 + 2.806^2} \approx 4.40$ km

Modelling and investigation

1 $\sin\theta = \frac{s}{1}$ $\cos\theta = \frac{c}{1}$ $\tan\theta = \frac{t}{1}$



2 Students' own diagrams.

Exercise 7.1

1 **a** 8.2 cm **b** 11.4 cm **c** 24 cm **d** 17.6 cm

2 **a** 28.8 cm^2 **b** 44.1 cm^2 **c** 23.0 cm^2 **d** 18.0 cm^2

3 **a** Area $= 3 \times 2 = 6$

b $AB = \sqrt{(3-1)^2 + (8-4)^2} = \sqrt{20}$

c Let the shortest distance be d.

$6 = \sqrt{20} \times d \Rightarrow d = \frac{6}{\sqrt{20}} = \frac{3\sqrt{5}}{5}$

Exercise 7.2

1 **a** 44.0 cm **b** 35.2 cm

2 **a** 9.00 cm **b** 7.50 cm

3 **a** 302 cm^2 **b** 6.16 cm^2

4 **a** 26.3 cm^2 **b** 75.3 cm^2

5 21.1 cm^2, 24.6 cm

6 **a** 5.17 cm **b** 2.79 cm

7 6.71 cm

8 12 cm

Exercise 7.3a

1 0.160 m^3, 160 l

2 362 250 cm^3, 39 950 cm^2

3 188 l

Investigation 7.1

1 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. Multipliers are 4, 9, 16, 25,… Multiplier is n^2. $\frac{9}{4} = \left(\frac{3}{2}\right)^2$

2 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000. Multipliers are 8, 27, 64, 125,… Multiplier is n^3. $\frac{27}{8} = \left(\frac{3^3}{2^3}\right)$

3 **a** 1.96 (approx. 2), 3.06 (approx. 3)

Exercise 7.3b

1 19.3 cm^3, 65.3 cm^2

2 309 cm^2

3 80.4 cm^3, 115 cm^2

4 14 cm^3, 39.3 cm^2

5 302 cm^3, 302 cm^2

6 345 cm^3, 238 cm^2

7 187500 m^3

8 61.3 cm^3

Chapter 7 test

1 circumference $= 2 \times \pi \times 3.8$
$= 23.9$ cm

1 mark + 1 for solution

area $= \pi \times 3.8^2$
$= 45.4$ cm^2

1 mark + 1 for solution

[4 marks]

2 circumference $= \pi \times 9.8$
$= 30.8$ cm

1 mark + 1 for solution

area $= \pi \times \left(\frac{9.8}{2}\right)^2$
$= 75.4$ cm^2

2 marks + 1 for solution

[5 marks]

3 $2 \times \pi \times r = 23.2$
$r = \frac{23.2}{2\pi}$
$r = 3.69$ cm

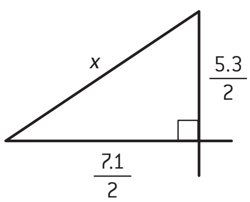
1 mark + 1 mark + 1 for solution

[3 marks]

4 $\pi r^2 = 12.7$ 1 mark + 1 mark + 1 for solution
$r^2 = \frac{12.7}{\pi}$
$r = \sqrt{\frac{12.7}{\pi}}$
$r = 2.01$
$d = 2 \times 2.01$ 1 mark
$d = 4.02$ cm

[4 marks]

5 1 mark



$x^2 = \left(\frac{5.3}{2}\right)^2 + \left(\frac{7.1}{2}\right)^2$ 2 marks
$x = \sqrt{\left(\frac{5.3}{2}\right)^2 + \left(\frac{7.1}{2}\right)^2}$
$x = 4.43$

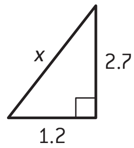
perimeter $= 4 \times 4.43$ 1 mark
$= 17.7$ cm

area $= \frac{1}{2} \times 5.3 \times 7.1$
$= 18.8$ cm^2

1 mark + 1 for solution

[6 marks]

6 $\frac{1}{2}(6.7 - 4.3) = 1.2$ 1 mark



$x^2 = 1.2^2 + 2.7^2$ 2 marks
$x = \sqrt{1.2^2 + 2.7^2}$
$x = 2.95$

perimeter $= 4.3 + 2 \times 2.95 + 2 \times 1.2 + \frac{1}{2}\pi \times 4.3$
$= 19.4$ cm

3 marks + 1 mark + 1 for solution

area $= \frac{1}{2}(4.3 + 6.7) \times 2.7 - \frac{1}{2}\pi \times \left(\frac{4.3}{2}\right)^2$
$= 7.59$ cm^2

3 marks + 2 marks + 1 for solution

[14 marks]

7 perimeter $= 3.5 + 1.5 + 1.5 + 4.3 + 1.5 + 4$
$= 16.3$ cm

2 marks + 1 for solution

area $= \frac{1}{2}(1.5 + 4) \times 3.5 + 4.3 \times 1.3$
$= 15.2$ cm^2

3 marks + 2 marks +1 for solution

[9 marks]

8 perimeter $= 2.9 + 5.6 + 5.6 + 2.9$
$= 17$ cm

3 marks + 1 for solution

area $= \frac{1}{2}(5 \times 6.5)$
$= 16.3$ cm^2

2 marks + 1 for solution

[7 marks]

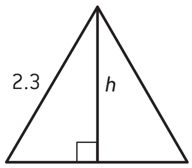
9 $V = 5.2 \times 15.2 \times 7.1$
$V = 561$ cm^3
$V = 561$ ml

Area $= 15.2 \times 7.1 + 2 \times 15.2 \times 5.2 + 2 \times 5.2 \times 7.1 \approx 340$ cm^2

2 marks + 1 + 1 for solution

[4 marks]

10



$$h^2 + \left(\frac{2.3}{2}\right)^2 = 2.3^2$$

$$h = \sqrt{2.3^2 - \left(\frac{2.3}{2}\right)^2}$$

$$h = 1.99$$

3 marks

a $V = \frac{1}{2} \times 2.3 \times 1.99 \times 8.2$

$V = 18.8 \text{ cm}^3$

3 marks + 1 for solution

b $A = 2 \times \frac{1}{2} \times 2.3 \times 1.99 + 3 \times 2.3 \times 8.2$

$A = 61.2 \text{ cm}^2$

2 marks + 2 marks + 1 for solution

[12 marks]

11 $V = (w \times 0.9) \times (t \times 0.95) \times l$

$V = wtl \times 0.855$

3 marks + 1 mark

reduction is 14.5 % 1 for solution

[5 marks]

12 volume $= \frac{2}{3} \times \pi \times 9.2^3 + \frac{1}{3} \times \pi \times 9.2^2 \times 15.6$

$= 3\,010 \text{ cm}^3$

3 marks + 3 marks + 1 for solution

[7 marks]

13 a $O\hat{B}C = 65°$

b $A\hat{B}O = \alpha$ (isosceles triangle)

$A\hat{O}B = 180 - 2\alpha$ (angles in a triangle sum to 180°)

$B\hat{O}C = 2\alpha$ (angles on a straight line)

$O\hat{B}C = \frac{180 - 2\alpha}{2} = 90 - \alpha$ (isosceles triangle)

Hence $A\hat{B}C = A\hat{B}O + O\hat{B}C$

$= \alpha + 90 - \alpha = 90°$

14 a Let d be the distance between the straights

$230 = \pi d \quad d \approx 73.2$ m

b Diameter of circle for the inside of the outside lane is $73.2 + 10 \times 1.2 = 85.2$

Hence total distance around the track is $170 + \pi \times 85.2 \approx 437.7$

So person in outside lane must begin $437.7 - 400 = 37.7$ m down the track.

15 a Slant height $= \sqrt{5_2 + 12_2}$

$= 13$ cm

Area of cone $= \pi \times 5 \times 13 = 65\pi$

b $\sin\theta = \frac{5}{13}$

c $\sin\theta = \frac{2}{x} = \frac{r}{x + 2 + r}$

d i $x = \frac{26}{5}$

$5(x + 2 + r) = 13r \quad 5x + 10 = 8r$

$5 \times \frac{26}{5} + 10 = 8r \quad r = \frac{9}{2} = 4.5$ cm

ii $V = \frac{4}{3} \times \pi \times \frac{9^3}{2^3} = \frac{243}{2}\pi$

e Distance to the top of the ice cream $\frac{26}{5} + 2 + 9 = 16.2$ cm

Hence 4.2 cm above the top of the cone.

Modelling and investigation

2 Circumferences A: 25.1 cm, B: 18.8 cm, C: 25.1 cm

3 It is only possible if the numbers are getting smaller and tending towards zero. This is illustrated in the example below.

4 a The area of the coloured rectangles are successively $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ so the sum of the areas of the coloured rectangles is given by the series given.

b The sum of the areas of the coloured rectangles is always less than the area of the large rectangle which is equal to 1.

5 a Using the formula $V = \pi r^2 h$

$$V = \pi \times 1^2 \times 1 + \pi \times \left(\frac{1}{2}\right)^2 \times 1 + \pi \times \left(\frac{1}{3}\right)^2 \times 1 + \ldots = \pi\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\right)$$

hence $a = \pi$

b Because every term on the right-hand side is greater than or equal to the corresponding term on the left-hand side.

c $V = \pi\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \ldots\right) < \pi\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\right)$

$< \pi(1 + 1) = 2\pi$ from part 4b

6 a Curved surface area of a cylinder is $2\pi rh$

Hence $A = 2\pi \times 1 \times 1 + 2\pi \times \frac{1}{2} \times 1 + 2\pi \times \frac{1}{3} \times 1 + \ldots$

$= 2\pi\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\right)$

So $b = 2\pi$

b Each term on the right hand side is smaller than or equal to the corresponding term on the left hand side.

c $A > 2\pi\left(1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \ldots\right)$

$= 2\pi\left(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots\right)$

If the series is continued to be divided as shown then it consists of $\frac{1}{2}$ being added an infinite number of times, which will eventually pass any finite number. As A is greater than this, it will also be greater than any finite number.

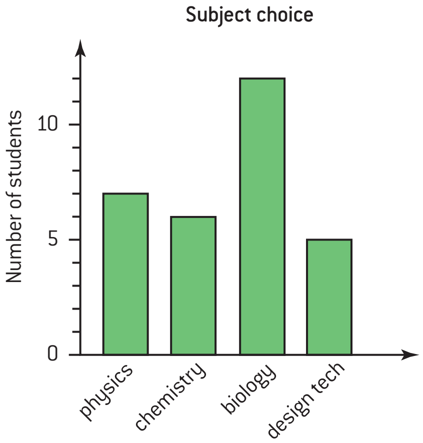
Answers – Chapter 8

Exercise 8.1a

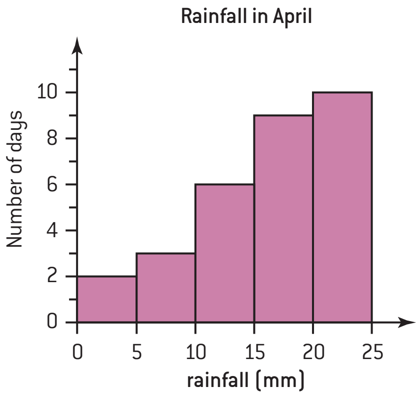
1 **a** 10 hot drinks, 16 cold drinks, 5 sandwiches, 19 ice creams;

b a warm day

2

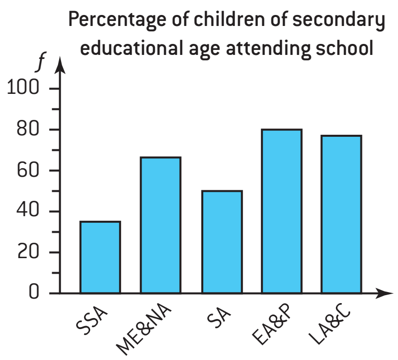


3



4 The sides are in the ratio 3:5 and hence the areas are in the ratio 9:25, which is close to 1:3. The diagram shows a symbol that is nearly 3 times the *area* so it is misleading.

5



6 **a** $1.5 \le x < 2$ **b** 16 **c** 44

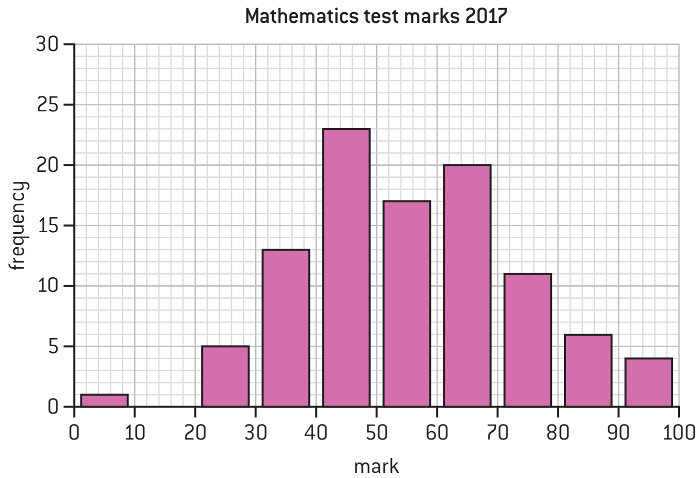
Investigation 8.1

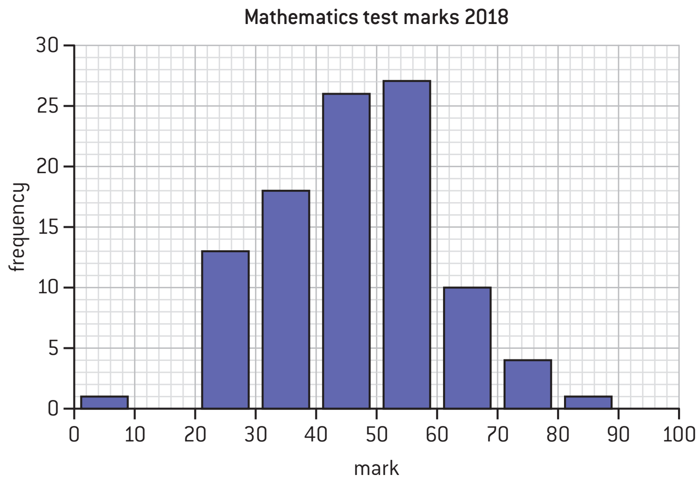
1

2017 results	
mark (m)	frequency
$0 \le m \le 9$	1
$10 \le m \le 19$	0
$20 \le m \le 29$	5
$30 \le m \le 39$	13
$40 \le m \le 49$	23
$50 \le m \le 59$	17
$60 \le m \le 69$	20
$70 \le m \le 79$	11
$80 \le m \le 89$	6
$90 \le m \le 100$	4

2018 results	
mark (m)	frequency
$0 \le m \le 9$	1
$10 \le m \le 19$	0
$20 \le m \le 29$	13
$30 \le m \le 39$	18
$40 \le m \le 49$	26
$50 \le m \le 59$	27
$60 \le m \le 69$	10
$70 \le m \le 79$	4
$80 \le m \le 89$	1
$90 \le m \le 100$	0

2

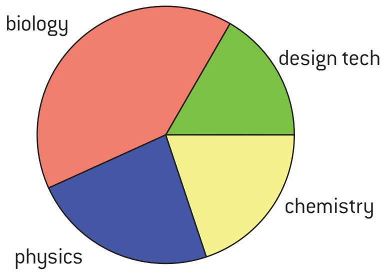




3 Low marks in both years are similar. 2017 has some higher marks. 2017 is more spread out. 2017 marks seem to be higher.

4 Results will vary according to the sample taken. Broadly the distribution will show some of the same features.

Exercise 8.1b

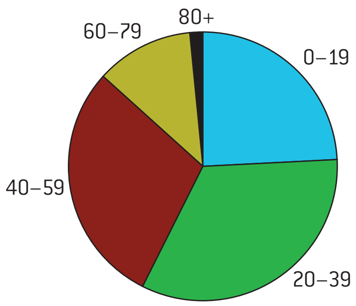


1 84°, 72°, 144°, 60°

2 1330 million, 0.271°

age	population (millions)	angle
0 – 19	320	87°
20 – 39	440	119°
40 – 59	390	106°
60 – 79	160	43°
80+	20	5°

age distribution of the population of China in 2010



3 **a** 44.4% **b** $45 **c** $270

Exercise 8.2a

1 **a** 5.2 **b** 9.42

2 **a** 4.2 **b** 5

3 3.25, 3, 2

4 32 340, 22 813, mean

5 100, 86.5

6 14.05, 14, 14.15, 14 median is unchanged

7 6.4

8 **a** 17 m **b** 9.3 m **c** 1.64 m

Investigation 8.2

1 58, 58, 59, 59, 59, 59, 59, 60, 60, 60, 60, 60, 60, 60, 60, 61, 61, 61, 61, 61

2 mean $= \dfrac{58+58+59+59+59+59+59+60+60+60+60+60+60+60+60+61+61+61+61+61}{20} = 59.8\text{g}$

median is 60g, mode is 60g.

3 mean $= \dfrac{(58\times 2)+(59\times 5)+(60\times 8)+(61\times 5)}{2+5+8+5}$

$= \dfrac{1196}{20}$

$= 59.8$

4 mode = 60

5 20, 10.5th, 60, median = 60,

Exercise 8.2b

1 **a** 2.02 cm **b** 2 cm **c** 2.5 cm

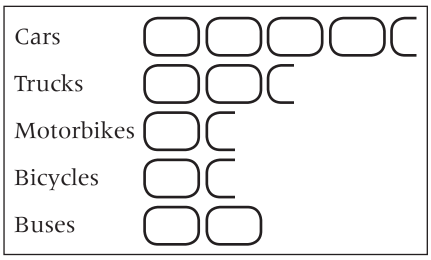
2 **a** $453 **b** $2.90
c $468, so would make $15 more

3 data is skewed by a small number of multi-millionaires

4 3

Chapter 8 test

1

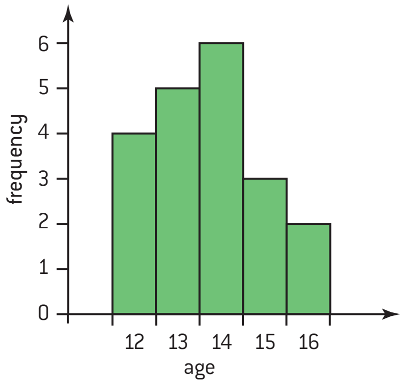


5 marks

2

Age	freq
12	4
13	5
14	6
15	3
16	2

5 marks



2 for axes 5 for bars

3

Day	Freq
Monday	3
Tuesday	2
Wednesday	1
Thursday	1
Friday	5

5 marks

Angles are Monday $= \frac{3}{12} \times 360 = 90°$,

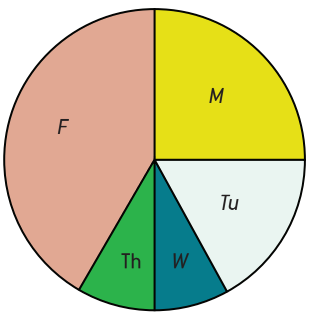
Tuesday $= \frac{2}{12} \times 360 = 60°$,

Wednesday $= \frac{1}{12} \times 360 = 30°$,

Thursday $= \frac{1}{12} \times 360 = 30°$,

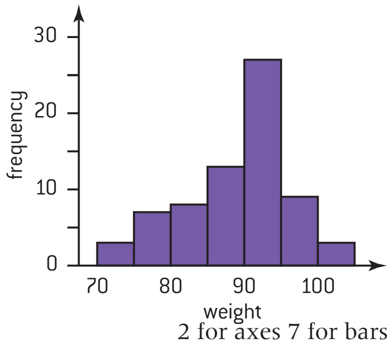
Friday $= \frac{5}{12} \times 360 = 150°$

5 marks



5 marks

4



2 for axes 7 for bars

[9 marks]

5 $\frac{84+72+93+85+77+78+89}{7} = \frac{578}{7}$

$= 82.6$ g

4 + 2 for result

6 The lengths are 46, 47, 48, 48, 48, 49, 50, 50, 51, 51, 52, 52, 53, 53, 54, 55 — 4 marks

The middle lengths are 50 and 51 — 2 marks

The median is 50.5 words — 1 mark

7 **a** 0, as four students have 0 coins. — 2 marks

b $16 - 0 = 16$. — 2 marks

8 $\frac{124+205+161+207+134+245+118+x}{8} = 171$ — 5 marks

$1194 + x = 1368$

$x = 174$ cm

9 **a** Total frequency is 14 so median is between the 7th and 8th.

These are both 3 so median is 3 hours — 2 marks

b mean $= \frac{0\times3+1\times2+2\times1+3\times5+4\times1+5\times2}{14}$ — 6 marks

$= \frac{33}{14}$

$= 2.36$ hours

Questions 1 – 9 out of 70

10 EITHER

This will be an underestimate as there were more games played by team A so their goals will contribute more to the overall total and hence the average.

OR

The total number of goals scored is $2.2 \times 10 + 1.8 \times 5 = 31$

Hence mean is $\frac{31}{15} = 2.07$ hence an underestimate. — 4 marks

b For example

how good the teams they have played against are.

What is the distribution of goals scored, have they mainly come from a couple of games or are they evenly distributed

Whether they have both played against the same team and what were the scores in that game.

How many goals each team let in

How many matches they have won / lost

Do either team have any players unavailable compared with previous games — 2 marks

11 **a** **i** small

ii medium — 2 marks

b $\frac{168}{360} \times 100 \approx 46.7\%$ — 2 marks

c $7 \equiv 28° \Rightarrow 1 \equiv 4°$

Small 42 Medium 21

Large 20 Total 90 — 3 marks

d If there were 4 more small dresses sold that would be 46 small dresses but the total would have risen to 94 and so the median would be the 47.5th dress. — 1 mark

$42 + x \geq \frac{90 + x + 1}{2}$

$\Rightarrow 84 + 2x \geq 91 + x \Rightarrow x \geq 7$

Smallest number is 7 (49 small out of 97 dresses) — 2 marks

Modelling and investigation

This is an open-ended task with no prescribed outcomes. You would expect to use bar charts and calculate the mean for under 20 and 20 and over groups.

Answers – Chapter 9

Exercise 9.1

1 $\frac{2}{5}$

2 $\frac{1}{10}$

3 **a** $\frac{3}{7}$ **b** 1

4 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{7}{12}$

5 0

6 $\frac{1}{2}$

7 **a** **i** 0.08 **ii** 0.4 **iii** 0.4

b Because they are not mutually exclusive as it could be warm and rain.

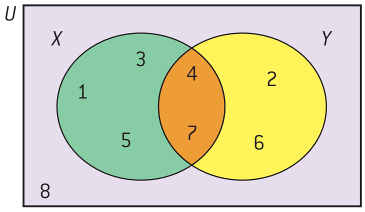
8 $\frac{19}{27}$

Exercise 9.2

1 $\frac{1}{4}$ **2** $\frac{1}{4}$ **3** $\frac{1}{6}$ **4** $\frac{1}{6}$

5 **a** $\frac{3}{10}$ **b** $\frac{1}{10}$ **c** $\frac{3}{10}$

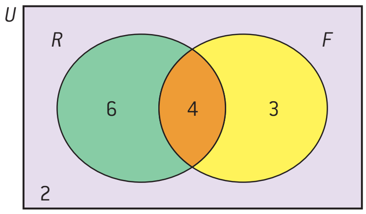
6



a $\{1, 3, 5, 8\}, \frac{1}{2}$ **b** $\{1, 3, 5\}, \frac{3}{8}$

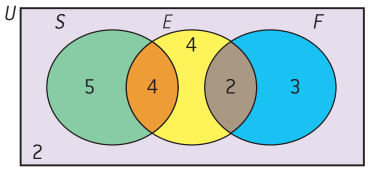
c $\{1, 2, 3, 4, 5, 6, 7\}, \frac{7}{8}$

7



a $\frac{2}{15}$ **b** $\frac{2}{5}$

8



a $\frac{1}{10}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{2}{5}$ **e** $\frac{3}{5}$

9 **a** $\frac{9}{20}$ **b** $\frac{13}{40}$ **c** $\frac{21}{40}$

d $\frac{2}{3}$ **e** $\frac{13}{19}$

Investigation 9.2

1 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{36}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$

3 $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

4 **a**

	Boy	Girl	Total
Has brother/sister	11	10	21
Does not have brother/sister	5	4	9
Total	16	14	30

b i $\frac{14}{30} = \frac{7}{15}$ ii $\frac{10}{14} = \frac{5}{7}$ iii $\frac{10}{30} = \frac{1}{3}$

c i $\frac{16}{30} = \frac{8}{15}$ ii $\frac{5}{16}$ iii $\frac{5}{30} = \frac{1}{6}$

d $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B \text{ given event } A)$

Exercise 9.3

1 **a** $\frac{4}{9}$ **b** $\frac{5}{9}$ **c** $\frac{4}{9}$

2 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$

3 **a** 0.216 **b** 0.064 **c** 0.288

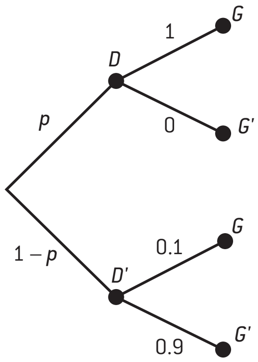
d 0.648

4 **a** $\frac{9}{20}$ **b** $\frac{9}{20}$ **c** $\frac{1}{10}$

5 **a** $\frac{11}{221}$ **b** $\frac{80}{221}$ **c** $\frac{10}{17}$

6 **a** $\frac{8}{27}$ **b** $\frac{19}{27}$ **c** $\frac{18}{27} = \frac{2}{3}$

7 **a** Let D be the event a fish has the disease and G the event the fish has grey patches.



b $p + (1 - p) \times 0.1 = 0.375$

$0.9p = 0.275 \Rightarrow p = \frac{11}{36} \approx 0.306$

Chapter 9 test

1 **a** $\frac{3}{6} = \frac{1}{2}$ 1 mark

b $\frac{3}{6} = \frac{1}{2}$ 1 mark

c 0 1 mark

d $1 - \frac{1}{6} = \frac{5}{6}$ 1 mark

[4 marks]

2 $\frac{2}{3}$ 2 marks

[2 marks]

3 $P(4, 5, 6) = \frac{3}{6} = \frac{1}{2}$ 3 marks

[3 marks]

4

	H	T
H	×	×
T	×	×

1 mark

a $\frac{2}{4} = \frac{1}{2}$ 1 mark

b $\frac{2}{4} = \frac{1}{2}$ 1 mark

[3 marks]

5

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

4 marks

a $\frac{9}{36} = \frac{1}{4}$ 1 marks

b $\frac{27}{36} = \frac{3}{4}$ 1 marks

[6 marks]

6

	1	1	1	3	6	6
1	2	2	2	4	7	7
1	2	2	2	4	7	7
1	2	2	2	4	7	7
3	4	4	4	6	9	9
6	7	7	7	9	12	12
6	7	7	7	9	12	12

4 marks

a $\frac{9}{36} = \frac{1}{4}$ 1 marks

b $\frac{6}{36} = \frac{1}{6}$ 1 marks

c $\frac{1}{36}$ 1 marks

d $\frac{12}{36} = \frac{1}{3}$ 1 marks

e $\frac{4}{36} = \frac{1}{9}$ 1 marks

f $\frac{4}{36} = \frac{1}{9}$ 1 marks

[10 marks]

7

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12

4 marks

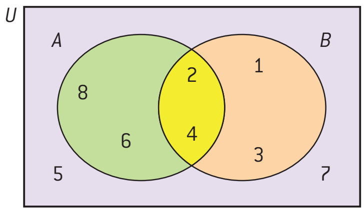
a P(10, 11, 12) = $\frac{6}{32} = \frac{3}{16}$ 1 mark

b P(2, 3, 4) = $\frac{3}{32}$ 1 mark

c P(6, 7, 8, 9) = $\frac{16}{32} = \frac{1}{2}$ 1 mark

[7 marks]

8



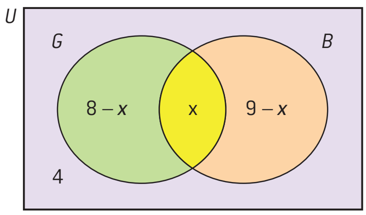
4 marks

a $\frac{2}{8} = \frac{1}{4}$ 1 mark

b $\frac{2}{8} = \frac{1}{4}$ 1 mark

[6 marks]

9



4 marks

$$4 + (8 - x) + x + (9 - x) = 16$$
$$21 - x = 16$$
$$x = 5$$

3 marks

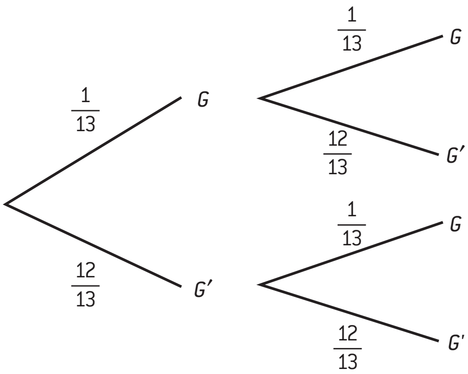
a $P(G \cap B) = \frac{5}{16}$ 1 mark

b $P(B \cap G') = \frac{1}{4}$ 1 mark

c $\frac{5}{8}$

[9 marks]

10



GG $P(GG) = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

GG' $P(GG') = \frac{1}{13} \times \frac{12}{13} = \frac{12}{169}$

$G'G$ $P(G'G) = \frac{12}{13} \times \frac{1}{13} = \frac{12}{169}$

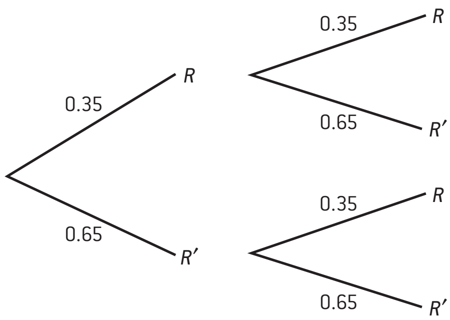
$G'G'$ $P(G'G') = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

6 marks

P(2 golds) = $\frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$ 1 marks

[7 marks]

11



RR $P(RR) = 0.35 \times 0.35 = 0.1225$

RR' $P(RR') = 0.35 \times 0.65 = 0.2275$

$R'R$ $P(R'R) = 0.65 \times 0.35 = 0.2275$

$R'R'$ $P(R'R') = 0.65 \times 0.65 = 0.4225$

6 marks

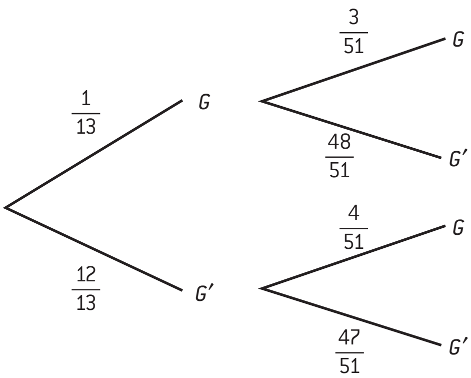
a $P(R\,R) = 0.35 \times 0.35 = 0.1225$ 1 marks

b $P(R\,R' \text{ or } R'\,R) = 0.35 \times 0.65 + 0.65 \times 0.35 = 0.455$ 1 marks

c $P(R'\,R') = 0.65 \times 0.65 = 0.4225$ 1 marks

[9 marks]

12



GG $P(GG) = \frac{1}{13} \times \frac{3}{51} = \frac{1}{221}$

GG' $P(GG') = \frac{1}{13} \times \frac{48}{51} = \frac{16}{221}$

$G'G$ $P(G'G) = \frac{12}{13} \times \frac{4}{51} = \frac{16}{221}$

$G'G'$ $P(G'G') = \frac{12}{13} \times \frac{47}{51} = \frac{188}{221}$

6 marks

P(2 golds) $= \frac{1}{13} \times \frac{3}{51} = \frac{1}{221}$ 2 marks

$\frac{1}{169} = 0.00592$ and $\frac{1}{221} = 0.00452$ 1 mark

The probability is less when the card is retained. 1 mark

[10 marks]

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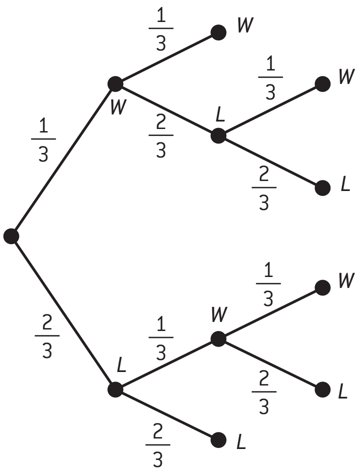
13 **a** $\frac{32}{80} = \frac{2}{5}$ **b** $\frac{3}{8}$ **c** $\frac{9}{25}$

14 **a** $\frac{6}{15} = \frac{2}{5}$ **b** $\frac{7}{15}$ **c** $\frac{12}{15} = \frac{4}{5}$ **d** $\frac{13}{15}$

15 $\frac{1}{14}$

16 **a**

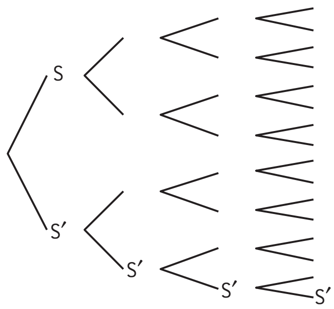


$\text{P}(\text{Cheryl wins}) = \frac{1}{9} + \frac{2}{27} + \frac{2}{27} = \frac{7}{27}$

c $\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$

Modelling and investigation

1 **a**



b $\text{P}(S) = \frac{1}{6}$ and $\text{P}(S') = \frac{5}{6}$

c getting S′ 4 times

d $\text{P}(S') = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

e $\text{P}(S) = 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

f 0.518

2 **a**

	1	2	3	4	5	6
1	×	×	×	×	×	×
2	×	×	×	×	×	×
3	×	×	×	×	×	×
4	×	×	×	×	×	×
5	×	×	×	×	×	×
6	×	×	×	×	×	⊗

$P(D') = \frac{35}{36}$

b getting D′ 24 times

c $\left(\frac{35}{36}\right)^{24}, 1 - \left(\frac{35}{36}\right)^{24}$

d 0.491

3 0.518 > 0.5 and 0.491

The probability of winning the game with two dice is $< \frac{1}{2}$, so he is more likely to lose than to win.

Exercise 10.1a

1 94.3 kmh^{-1} **2** 687 kmh^{-1}
3 215 km **4** 1 h 24 min
5 8.22 cm/day **6** 13.9 m/s
7 38.0 km/h

8 **a** $\frac{40}{0.4} = 100$ seconds

b 30 m

c distance travelled $= \sqrt{40^2 + 30^2} = 50\text{m}$ so 0.5 ms^{-1}.

d $\tan^{-1}\left(\frac{0.4}{0.3}\right) = 53.1°$

e Let the distance downstream be x.
Then $\frac{30}{x} = \tan 53.1°$
So $x = \frac{30}{\frac{4}{3}} = \frac{90}{4} = 22.5\text{m}$

f Philipp has travelled 30m so time taken $= \frac{30}{0.4} = 75$ seconds
$\frac{10}{75} \approx 0.133$ ms^{-1}.

g $\cos^{-1}\left(\frac{0.3}{0.4}\right) = 41.4°$

h Each second he will travel $0.4\sin(41.4°) = 0.264...$ across the river
Time $= \frac{10}{0.264...} \approx 37.8$ seconds

Investigation 10.1a

1 50 km **2** 75 km/h **3** stationary
4 1 h 40 min to 2 h **5** 15 km/h
6 120 km/h

Exercise 10.1b

1 **a** 0.4 ms^{-1} **b** 2 kmh^{-1}
c 13.3 kmh^{-1} **d** 0.35 ms^{-1}

2 **a** 2.5 ms^{-1} **b** 0 ms^{-1} **c** 1 ms^{-1}
d 0 ms^{-1} **e** 2.5 ms^{-1} **f** 1 ms^{-1}

3 **a** t = 1 to t = 1.6, t = 2.6 to t = 3
b t = 0 to t = 1, t = 1.6 to t = 2.6, t = 4 to t = 5 **c** 20 km/h
d 10 km/h

Exercise 10.2

1 2,470,000

2 3.33 $kmh^{-1}s^{-1}$

3 **a** 1.1375 ls^{-1}

b 1 hour 43 minutes

4 **a** Jan-March 6 cm $month^{-1}$

b July – Sept 12.5 cm $month^{-1}$

c 1 cm increase

d 32 cm

Chapter 10 test

1 5 hours 34 minutes $= 5\frac{34}{60}$ hours
$= 5.566...$ hours — 1 mark

speed $= \frac{\text{distance}}{\text{time}}$ — 3 marks
$= \frac{534}{5.566...}$
$= 95.9$ km/h

[4 marks]

2 distance = speed × time — 3 marks
$= 80 \times 4.5$
$= 360$ km

[3 marks]

3 1 light year =
$= 300000 \times 60 \times 60 \times 24 \times 365$
$= 9.46 \times 10^{12}$ km

3 marks

[3 marks]

4 time $= \frac{\text{distance}}{\text{speed}}$ — 4 marks
$= \frac{1.5}{5.1}$
$= 0.294$ hours
$= 0.294 \times 60$
$= 18$ min

[4 marks]

5 speed $= \frac{100}{23}$ — 5 marks
$= 4.35$ m/s
$= \frac{4.35 \times 3600}{1000}$
$= 15.7$ km/h

[5 marks]

6 **a** $\frac{12}{10} = 1.2$ ms^{-1} — 2 marks

b $\frac{10}{4} = 2.5$ ms^{-1} — 2 marks

c 2 sec — 1 mark

d $\frac{12-10}{4} = \frac{2}{4}$ — 2 marks
$= 0.5$ ms^{-1}

[7 marks]

Questions 1 – 6 out of 26 marks

7 **a** $\frac{670}{320} = 2.09$ hours or
2 hours 6 minutes — 2 marks

b $\frac{670}{2\frac{55}{60}} = 230$ kmh^{-1} — 3 marks

[5 marks]

8 **a** $0.2t$ and $0.4t$ — 2 marks

b $0.2t + 0.4t = 1.2$
$t = 2$ seconds — 2 marks
Distance from
$A = 2 \times 0.2 = 0.4\text{m}$ — 1 mark

[5 marks]

9 Volume of full tank =
$\frac{2.0}{2}(1.8 + 2.2) \times 2.8 = 11.2$ m^3

2 marks

When height of oil is 0.5 m the width of the trapezium is 1.9 m — 2 marks

Volume of oil =
$\frac{0.5}{2}(1.8 + 1.9) \times 2.8 = 2.59$ m^3

1 mark

Time $= \frac{11200 - 2590}{50} = 172.2$ days

2 marks

[7 marks]

10 $v_2 = 3v_1, \quad t_1 + t_2 = 2$ — 1 mark

$v_1 t_1 = 10$ — 1 mark

$8 = \frac{v_1 t_1 + v_2 t_2}{2}$ — 1 mark

$16 = v_1 t_1 + 3v_1(2 - t_1)$ — 1 mark

$16 = 6v_1 - 2v_1 t_1$

$16 = 6v_1 - 20 \quad v_1 = 6$ — 1 mark

$v_2 = 18, \; t_1 = \frac{10}{6} \approx 1.67 \; t_2 = \frac{1}{3} \approx 0.33$

2 marks

[7 marks]

Modelling and investigation

1 **a** $2 \times 2 + 2 \times 3 = 10$ m

b an underestimate

c By taking the value for the speed at the middle of the interval rather than at the end

By having more rectangles of smaller width.

2 The distance travelled is
$\frac{1}{2}(2 + 4) \times 4 = 12$ m

3 **a** If using lower rectangles: $0\times1+0.2\times1+0.8\times1+1.8\times1+3.2\times1=6.0$ m

If using upper rectangles: $0.2\times1+0.8\times1+1.8\times1+3.2\times1+5\times1=11.0$ m

If using the mid-points distance is 8.5 m

b If using lower rectangles, underestimate; if using upper rectangles, overestimate; if using mid-points, ambiguous.

c Increase the number of rectangles.

4 Exact area $=\frac{25}{3}\approx 8.33$